

# HW4

*October 8, 2026*

**Reminder:** Submit your assignment on Gradescope by the due date. Submissions must be typeset. Each page should include work for only one problem (i.e., make a new page/new pages for each problem). See the course syllabus for the late policy.

While collaboration is encouraged, please remember not to take away notes from any labs or collaboration sessions. Your work should be your own. Use of other third-party resources is strictly forbidden.

Please monitor Ed discussion, as we will post clarifications of questions there.

## Problem 1

1. Prove that the set of Context-Free Languages is not closed under the complement operator.
2. Prove that the set of Deterministic Context-Free Languages is not closed under the union operator.
3. If  $A$  and  $B$  are languages, define

$$A \circ B = \{xy \mid x \in A \text{ and } y \in B \text{ and } |x| = |y|\}.$$

Given  $A$  and  $B$  are regular languages, argue whether  $A \circ B$  is a CFL.

## Problem 2

Let

$$L_1 = \{w \in 1, \#^* \mid w = t_1 \# t_2 \# \dots \# t_k \text{ for } k \geq 0, \text{ each } t_i \in 1^*, \text{ and } t_i \neq t_j \text{ whenever } i \neq j\}.$$

- Prove that  $L_1$  is not a Context-Free language.
- Prove that  $L_1$  is a Turing-Decidable language. If you choose to do so by showing an accepting Turing Machine  $M_1$  such that  $L_1 = L(M_1)$ , you need to show a proof of correctness.

**Problem 3**

Let  $L_2 = \{w \in \{0, 1\}^* \mid w = w^R \text{ and there are the same number of 0's as 1's in } w\}$ .

- Prove that  $L_2$  is not a Context-Free language.
- Prove that  $L_2$  is a Turing-Decidable language. If you choose to do so by showing an accepting Turing Machine  $M_1$  such that  $L_1 = L(M_1)$ , you need to show a proof of correctness.