CS 170 Homework 6

Due **3/7/2023, at 10:00 pm (grace period until 11:59pm)**

1 Study Group

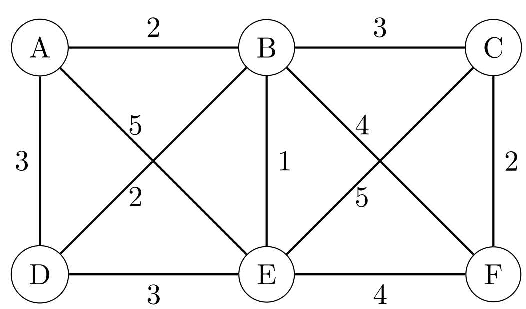
List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write "none".

2 Prim's Algorithm

A popular alternative to Kruskal's algorithm is Prim's algorithm, in which the intermediate set of edges T always forms a subtree, and S is chosen to be the set of this tree's vertices. We can think of Prim's algorithm as greedily processing one vertex at a time, adding it to S. The pseudocode below gives the basic outline of Prim's algorithm. See the DPV textbook for a detailed example of a run of the algorithm.

procedure PRIMS($G = (V, E)$)
  $S \leftarrow \{v\}$
  $T \leftarrow \{\}$
  while $S \neq V$ **do**
    Choose $s \in S$ and $t \in V \setminus S$ such that $w(s, t)$ is minimized
    $T \leftarrow T \cup \{(s, t)\}$
    $S \leftarrow S \cup \{t\}$
  return T

(a) Run Prim's algorithm on the following graph, starting from A, stating which node you processed and which edge you added at each step.



(b) Prim's algorithm is very similar to Dijkstra's in that a vertex is processed at each step which minimizes some cost function. These algorithms also produce similar outputs; the union of all shortest paths produced by a run of Dijkstra's algorithm forms a tree as well. However, the trees that Prim's and Dijkstra's produce aren't optimizing for the same thing. To see this, give an example of a graph for which different trees are produced by running Prim's algorithm and Dijkstra's algorithm. In other words, give a graph where there is a shortest path from a start vertex A using at least one edge that doesn't appear in any MST.

3 Second Best MST

For any undirected graph G with edge weight function c, the second best minimum spanning tree of G is defined to be the spanning tree with the second minimum weight, among all spanning trees of G. Note that in certain graphs the second best MST may have the same weight as the MST.

Design an algorithm that, upon input $G = (E, V)$ with edge weight function w, finds the second best MST. (When the second best MST is not unique, ties can be broken arbitrarily.)

Give a 3-part solution.

4 Rigged Tournament

Siqi is in charge of organizing a basketball tournament with n teams. The tournament is a single-elimination tournament: if teams i and j play, the team that loses is out of the tournament and cannot play any more games. There are no ties.

Siqi's shady friend Jonny has given her the following inside information: if teams i and j play, then they will score a combined total of $p(i, j) \geq 0$ points in that game and furthermore Jonny can rig the match so that the team of her choice wins. Siqi wishes to find a tournament schedule which (1) maximizes the number of points scored in the tournament, and (2) makes her favorite team, team s, win the tournament. Give an efficient algorithm to solve this problem; **proof of correctness and runtime analysis are not needed.**

Note: teams need not play an equal number of games in the tournament. For example, if the teams are $\{1, 2, 3, 4\}$, then a valid tournament schedule (where (i, j) means i plays j and wins) is $((1, 2), (1, 3), (4, 1))$. Here, team 4 wins the tournament.