

HW5

10/15/2025 at 10:20am

Reminder: Submit your assignment on Gradescope by the due date. Submissions must be typeset. Each page should include work for only one problem (i.e., make a new page/new pages for each problem). See the course syllabus for the late policy.

While collaboration is encouraged, please remember not to take away notes from any labs or collaboration sessions. Your work should be your own. Use of other third-party resources is strictly forbidden.

Please monitor Ed discussion, as we will post clarifications of questions there.

Problem 1

A *Turing machine with left reset* is similar to an ordinary Turing machine, but the transition function has the form:

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{R, RESET\}$$

.

If $\delta(q, a) = (r, b, RESET)$, when the machine is in state q reading an a , the machine's head jumps to the left-hand end of the tape after it writes b on the tape and enters state r . Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

Problem 2

Given an arbitrary Turing decidable language L' , consider the following language

$$L = \{w | \exists w' \in L' \text{ such that } w \text{ is a substring of } w'\}$$

1. Prove that L is Turing recognizable. That is, give a TM recognizer M for L . Prove that it is correct (i.e., $L = L(M)$) and that it halts and accepts any member of L

2. Without providing rigorous proof, give an intuitive argument for whether you believe L to be decidable.
3. Suggest a property that, if L' were to have it, would guarantee L to be decidable. Motivate your answer.

Problem 3

A *very sparse language* is a language L over an alphabet Σ that contains exactly one string of each length. If L is a very sparse language, it contains the empty string, exactly one string of length one, exactly one string of length two, and so on. Prove that if L is a very sparse language and is Turing-recognizable, then L is decidable.