CSCI 1010 - Fall'26 Theory of Computation Lorenzo De Stefani

HW8

November 12, 2026 at 11:59 pm

Reminder: Submit your assignment on Gradescope by the due date. Submissions must be typeset. Each page should include work for only one problem (i.e., make a new page/new pages for each problem). See the course syllabus for the late policy.

While collaboration is encouraged, please remember not to take away notes from any labs or collaboration sessions. Your work should be your own. Use of other third-party resources is strictly forbidden.

Please monitor Ed discussion, as we will post clarifications of questions there.

Problem 1

Let ϕ be Boolean Formula in CNF, and let $ALLBUT2SAT = \{< \phi > \mid \exists \phi'$ which includes all the clauses of ϕ except exactly two, and which is satisfiable $\}$. Show that $ALLBUT2SAT$ is NP-complete.

Problem 2

An organization has a finite set P of participants and a collection of projects $\mathcal{G} = \{G_1, \ldots, G_m\}$, where each project $G_i \subseteq P$ contains between two and four participants.

The organization wishes to assign every participant to exactly one of two teams, Team A or Team B, subject to the following requirement:

> Every project must include at least one participant from each team.

A participant may belong to multiple projects, but must be assigned to the same team across all projects.

Define the language

$$\textsc{Two-Team-Coverage} = \left\{ \langle P, \mathcal{G} \rangle \;\middle|\; \begin{array}{l} \text{there exists an assignment of participants} \\ \text{to Teams A and B satisfying every project} \end{array} \right\}.$$

Prove that Two-Team-Coverage is NP-complete by addressing the following:

(a) Show that Two-Team-Coverage $\in$ NP.

(b) Construct a polynomial-time reduction from 3SAT to Two-Team-Coverage.

(c) Prove that your reduction is correct in both directions, and justify its polynomial running time.

Hint: Consider a formula in 3SAT with variables $x_1, \ldots, x_n$. For each variable x_i, introduce two participants, one representing the literal x_i and the other representing $\neg x_i$.

First, think about how to create a project that forces these two participants to belong to opposite teams. This will allow you to interpret their team assignments as opposite truth values.

Next, consider introducing one additional participant who appears in every project corresponding to a clause. Think of the team containing this special participant as representing *false*, and the other team as representing *true*. How can you use this idea to construct a project for each clause that satisfies the two-team requirement if and only if the clause contains at least one true literal?

Remember that the same participant should be reused whenever the corresponding literal appears in multiple clauses.

Problem 3

Consider a bipartite graph $G = (L, R, E)$. We would like to color each vertex in L either red or blue. A coloring is considered valid if every vertex in R has at least one red neighbor and at least one blue neighbor. Let $c(v)$ denote the color assigned to vertex v.

Formally, define

Bichromatic-Neighborhoods $= \{\langle G, L, R\rangle$ *s.t.* $G = (L, R, E)$ *is a bipartite graph and there exists a coloring of its verices such that for every* $v \in R$, *there exist* $u, w \in L$ *with* $(u, v), (w, v) \in E$, $c(u) =$*red and* $c(w) =$*blue* $\}$.

Prove that Bichromatic-Neighborhoods$\in NP$ by showing a correct polynomial time verifier.