

HW1

Due on 9/17/2026 at 10:20am

Reminder: Submit your assignment on Gradescope by the due date. Submissions must be typeset. Each page should include work for only one problem (i.e., make a new page/new pages for each problem). See the course syllabus for the late policy.

While collaboration is encouraged, please remember not to take away notes from any labs or collaboration sessions. Your work should be your own. Use of other third-party resources is strictly forbidden.

Please monitor Ed discussion, as we will post clarifications of questions there.

Problem 1

Prove that for any integer $k > 0$, there is a language L such that

- (a) L can be recognized by a DFA with k states
- (b) No DFA with less than k states recognizes L

Problem 2

Design a DFA that recognizes the following language over the alphabet $\{0, 1\}$. Provide a proof of correctness.

$\{w \mid w \text{ contains an even number of 0s and the substring } 010\}$

Problem 3

- (a) Let $A \subseteq \Sigma^*$ be a regular language, and let

$$M = (Q, \Sigma, \delta, q_0, F)$$

be a DFA recognizing A .

Define a new DFA

$$M' = (Q, \Sigma, \delta, q_0, Q \setminus F).$$

What language does M' recognize? Prove your answer.

What general conclusion can you draw from this construction about regular languages?

- (b) Consider the language

$$L = \{w \in \{a, b\}^* \mid w \text{ has an odd number of } b\text{'s and contains the substring } ab\}.$$

Write L as the intersection

$$L = L_1 \cap L_2$$

of two simpler languages. Clearly define L_1 and L_2 .

- (c) Construct a DFA recognizing L_1 and a DFA recognizing L_2 .

For each DFA, prove that it recognizes the intended language.

- (d) Assume that you have already proved that regular languages are closed under union, that is, the union of two regular languages is regular. Argue that L is regular.