ME 103 Experimentation and Measurements
Spring 2026 George Anwar
HW2

Due: Monday, February 23rd at 11:59 pm

- This homework will cover the hypothesis testing and an introduction to frequency response methods and spectral analysis.
- In all of the questions, **show your work**, not just the final answer. Unless we explicitly state otherwise, you may expect full credit only if you explain your work succinctly, but clearly and convincingly. For coding questions, attach a screenshot of your code and output.
- Present your answers with **a suitable number of significant figures** for each question. Show your work, including a mathematical formula or the MATLAB or Python code you wrote, before reaching the result. You may need to install the Statistics Toolbox if using MATLAB.
- Throughout this assignment, neglect systematic (bias) errors. Also, assume a normal distribution for the underlying distribution (population) if necessary.
- If you have a confirmed disability that precludes you from complying fully with these instructions or with any other parameter associated with this problem set, please alert us immediately about reasonable accommodations afforded to you by the DSP Office on campus.
- **Start early. Some of the material is prerequisite material not covered in lecture; you are responsible for finding resources to understand it.**

Deliverables

Submit a PDF of your homework to the **Gradescope assignment** entitled "{Your Name} HW2". You **must** typeset your homework in LaTeX (submit PDF format, not .doc/.docx format) . Mac Preview, PDF Expert, and FoxIt PDF Reader, among others, have tools to let you sign a PDF file. We want to make *extra clear* the consequences of cheating.

0 Honor Code

I will adhere to the Berkeley Honor Code: specifically, as a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. Failure to comply with these guidelines can be considered an academic integrity violation. Please email Professor Anwar ganwar@berkeley.edu if you have any questions!

- **List all collaborators. If you worked alone, then you must explicitly state so. Read the following statement and sign below if you agree**:

  "I certify that all solutions in this document are entirely my own and that I have not looked at anyone else's solution. I have given credit to all external sources I consulted."

  Signature : _____ *Date* : _____

  While discussions are encouraged, *everything* in your solution must be your (and only your) creation. Furthermore, all external material (i.e., *anything* outside lectures and assigned readings, including figures and pictures) should be cited properly. We wish to remind you that consequences of academic misconduct are *particularly severe*!

- Violation of the Code of Conduct will result in a **zero** on this assignment and may also result in disciplinary action.

1 Hypothesis Testing Basics [18 points]

(a) [9 pts] A food manufacturer claims its ready-meals contain on average no more than 2.0g of salt per serving. From a sample of 30 servings the mean salt content is found to be 2.2g. You may assume that the variance is known to be 0.2. Use an appropriate hypothesis test to determine whether the claim is correct. You may use a significance level of 2.5%.

Solution: TODO

(b) [9 pts] 26 students who attended a course in-person obtained an average score of 60% in an exam, the standard deviation of their scores was 10%. There were an additional 36 students who attended online only, these students obtained an average score of 55% with a standard deviation of 12%. Is there a difference in the average exam scores between these two groups? You may use a significance level of 5.0% and you may assume equal variance.

Solution: TODO

2 Non-Parametric Statistics [19 points]

(a) [3 pts] Briefly describe two advantages and one disadvantage of using non-parametric tests.

Solution: TODO

(b) [8 pts] A business contracts two parcel delivery companies (Company A and Company B) to deliver its goods. This business obtains feedback about how satisfied each customer was with their delivery. Satisfaction is rated on a scale of 0 to 20, the higher the rating the more satisfied the customer.

Company	**Satisfaction**
B	9
A	10
A	11
B	15
A	17
B	19
B	20
A	20

Is there a difference in the median customer satisfaction between the deliveries made by these two companies? Use an appropriate non-parametric test to answer this question. You may use a significance level of 5%.

Solution: TODO

(c) [8 pts] Six students rate two statistics textbooks on a scale from 1 to 10, where 1 is the easiest to understand and 10 being the hardest. The books are *All of Statistics* and *Bayesian Data Analysis*. Use an appropriate non-parametric test to determine whether students find the book *All of Statistics* harder to understand than *Bayesian Data Analysis*. You may use a significance level of 5%.

Student	**All of Statistics**	**Bayesian Data Analysis**
1	3	9
2	5	1
3	2	4
4	7	2
5	1	8
6	8	3

Solution: TODO

3 Hypothesis Testing for two samples [22 points]

Two different formulations of an elastomeric material are being evaluated, and Young's moduli at small strains are measured from a sample of pieces of each material. One formulation ('Treatment 1') results in a sample mean of 2.085 MPa while the other ('Treatment 2') results in a sample mean of 2.071 MPa. The sample standard deviations are 25 kPa and 34 kPa respectively.

As you carry out the following computations, show your working for each part and summarize your final computed results in the table on the next page (a Word version is available on bCourses if you wish to edit it).

(a) [3 points] Consider first the case (Case A in the table) where the above data are obtained from sample sizes of 15 measurements for each treatment. Calculate the effective number of degrees of freedom using the following formula:

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Solution: TODO

(b) [3 points] Calculate the t-statistic for the difference between the sample means, i.e.:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

Solution: TODO

(c) [3 points] Using tables or software, find the critical t-values for:

(i) A 2-tailed t-test at the 5% significance level where the null hypothesis is $H_0 : \mu_1 = \mu_2$ and the alternate hypothesis is $H_a : \mu_1 \neq \mu_2$. The values μ_1 and μ_2 are the population mean Young's moduli resulting from treatments 1 and 2 respectively.

Solution: TODO

(ii) A 1-tailed t-test at the 5% significance level where the null hypothesis is $H_0 : \mu_1 = \mu_2$ and the alternate hypothesis is $H_a : \mu_1 > \mu_2$.

Solution: TODO

(d) [3 points] Evaluate the two hypothesis tests described in 1.3(a) and 1.3(b): is the null hypothesis accepted or rejected in each case?

Solution: TODO

(e) [6 points] Now assume that the sample means and standard deviations are still as given above, but instead they were calculated from smaller and unequal sample sizes: $n_1 = 7$ for treatment 1 and $n_2 = 5$ for treatment 2 (this is Case B in the table below). Recalculate the effective degrees of freedom and t-values and re-evaluate the two hypothesis tests (1- and 2-tailed) assuming these smaller sample sizes.

Solution: TODO

(f) [3 points] Comment on the impact of the sample sizes on the conclusions that are drawn in the hypothesis tests.

Solution: TODO

(g) [1 point] Fill in the summary table on the next page.

	Treatment 1	**Treatment 2**
Sample mean, $\bar{x}$ (MPa)	2.085	2.071
Sample standard deviation, S_x (MPa)	0.025	0.034
Case A		
Sample size	15	15
Effective degrees of freedom	TODO	
t statistic calculated for data	TODO	
t_{crit}, 2-tailed test where $H_0 : \mu_1 = \mu_2$ and $H_a : \mu_1 \neq \mu_2$	TODO	
Accept or reject null hypothesis for 2-tailed test?	TODO	
t_{crit}, 1-tailed test where $H_0 : \mu_1 = \mu_2$ and $H_a : \mu_1 > \mu_2$	TODO	
Accept or reject null hypothesis for 1-tailed test?	TODO	
Case B		
Sample size	7	5
Effective degrees of freedom	TODO	
t statistic calculated for data	TODO	
t_{crit}, 2-tailed test where $H_0 : \mu_1 = \mu_2$ and $H_a : \mu_1 \neq \mu_2$	TODO	
Accept or reject null hypothesis for 2-tailed test?	TODO	
t_{crit}, 1-tailed test where $H_0 : \mu_1 = \mu_2$ and $H_a : \mu_1 > \mu_2$	TODO	
Accept or reject null hypothesis for 1-tailed test?	TODO	

4 Nyquist Criterion and Frequency Resolution [13 points]

(a) [3 points] First, suppose that it can be confidently assumed that there is no frequency content in the accelerometer's output signal that is above 2 kHz. What is the minimum sample rate that should be used for the measurements?

Solution: TODO

(b) [3 points] To reliably test the model of the machine's vibration, the peak resonant frequency must be resolved to ± 1 Hz. How can this level of resolution be achieved? How long will the measurement take to complete if the minimum number of samples is recorded?

Solution: TODO

(c) [5 points] Next, suppose that the accelerometer may be sensing some additional harmonics above 2 kHz. To avoid the possibility of aliasing, a first-order low-pass filter is connected between the accelerometer and the A2D board. The -3 dB frequency of the filter is set to 2.5 kHz. The filter can be assumed to cut off signals completely if they occur at frequencies where the gain of the filter is 0.05 or less. What should the sampling rate now become to avoid aliasing?

Solution: TODO

(d) [2 point] Comment on your result in part 3 and whether there may be a better alternative to a first-order low-pass filter for this experiment.

Solution: TODO

5 Manifestation of aliasing [12 points]

A 500 Hz sine wave is sampled at a frequency of 4096 Hz. A total of 2048 points are taken.

(a) [4 point] Calculate the Nyquist frequency for this sampling process.

Solution: TODO

(b) [4 point] Calculate the frequency resolution of the sampling process.

Solution: TODO

(c) [4 point] Suppose that the sampled waveform contains several harmonics of 500 Hz (i.e., signals at integer multiples of 500 Hz). Which of these can be accurately measured? What happens to the others?

Solution: TODO

6 Manifestation of Aliasing [16 points]

A 150 Hz cosine wave is sampled at a rate of 200 Hz.

(a) [6 points] Draw the wave and the temporal locations at which it is measured. You might want to use or modify the Matlab code below to accomplish this:

```
% Define duration T that is 20 sample periods
T = 20/200;
% Define time vector with 1000 points
t = linspace(0,T,1000);
% Generate cosine wave signal at frequency 150 Hz
B = cos(2*pi*150*t);
% Plot the signal waveform
plot(t,B)
% Generate time series of samples, t1
t1 = 0:(1/200):T;
% Evaluate values of 150 Hz wave at the sampling intervals
B1 = cos(2*pi*150*t1);
hold on
% Plot the samples measured
plot(t1,B1,'ro')
```

Solution: TODO

(b) [4 point] What apparent frequency is measured?

Solution: TODO

(c) [6 points] Provide an explanation for the numerical relationship between the actual signal frequency (150 Hz) and the measured frequency.

Hint: Look at material on heterodyning, and imagine that the 'carrier' wave has frequency 200 Hz and is multiplied by the signal wave, a sinusoid of 150 Hz. What output frequency/frequencies does the model for heterodyning predict?

Solution: TODO