CS 170 Homework 4

Due **2/21/2022, at 10:00 pm (grace period until 11:59pm)**

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write "none".

2 Infinite Paths

Let $G = (V, E)$ be a directed graph with a designated "start vertex" $s \in V$ and a set $V_G \subseteq V$ of "good" vertices . An infinite trace p of G is an infinite sequence $\{v_0, v_1, v_2, \dots\}$ of vertices $v_i \in V$ such that

1. $v_0 = s$, and
2. For all $i \geq 0$, $(v_i, v_{i+1}) \in E$.

That is, p is an infinite path in G starting at vertex s. Since the set V of vertices is finite, every infinite trace of G must visit some vertices infinitely often.

(a) If p is an infinite trace, let $\mathrm{Inf}(p) \subseteq V$ be the set of vertices that occur in p infinitely often. Show that $\mathrm{Inf}(p)$ is a subset of a strongly connected component of G.

(b) Describe an algorithm that determines if G has an infinite trace (note that any trace must start from vertex s). Provide a 3-part solution.

(c) Describe an algorithm that determines if G has an infinite trace that visits some good vertex in V_G infinitely often. You only need to provide a description of the algorithm; proof of correctness and runtime analysis are not required.

3 Bounded Bellman-Ford

Modify the Bellman-Ford algorithm to find the length of the shortest path from s to t with the restriction that the path must have at most k edges. Just a description of your modification is needed; no proof of correctness nor runtime analysis is required.

4 The Greatest Roads in America

Arguably, one of the best things to do in America is to take a great American road trip. And in America there are some amazing roads to drive on (think Pacific Coast Highway, Route 66 etc). An intrepid traveler has chosen to set course across America in search of some amazing driving. What is the length of the shortest path that hits at least k of these amazing roads?

Assume that the roads in America can be expressed as a directed weighted graph $G = (V, E, w)$, and that our traveler wishes to drive across at least k roads from the subset $R \subseteq E$

of "amazing" roads. Furthermore, assume that the traveler starts and ends at her home $a \in V$. You may also assume that the traveler is fine with repeating roads from R, i.e. the k roads chosen from R need not be unique.

Design an algorithm that solves this problem in $O(k(|E| + |V|)\log(k|V|))$ time. Provide a 3-part solution.

5 Preventing Conflict

A group of n guests shows up to a house for a party, but m pairs of these guests are enemies (a guest can be enemies with multiple other guests). There are two rooms in the house, and the host wants to distribute guests among the rooms, breaking up as many pairs of enemies as possible. The guests are all waiting outside the house and are impatient to get in, so the host needs to assign them to the two rooms quickly, even if this means that it's not the best possible solution.

Come up with a $O(n + m)$-time algorithm that breaks up at least half of the pairs of enemies. Provide a 3-part solution.

6 (Extra Credit) Coding Question

For this week's homework, you'll have the option to implement your solution to Q4 (The Greatest Roads in America) in the python jupyter notebook called `greatest_roads.ipynb`. There are two ways you can access the notebook and complete the problems:

1. Click here and navigate to the `HW4` folder if you prefer to complete this question on Berkeley DataHub.

2. Run

   ```
   git clone https://github.com/Berkeley-CS170/cs170-coding-notebooks-sp23
   ```

   in your computer's terminal (and navigate to the `HW4` folder) if you prefer to complete it locally. If you run into any issues with python import or autograder errors, please refer to the local setup instructions here.

Notes:

- *This week's coding homework is extra credit!* It will be worth around 10% of the total points on the homework (e.g. out of 30 pts, this coding portion would be worth +3 pts extra credit).

- *Submission Instructions:* Please download your completed `greatest_roads.ipynb` file and submit it to the gradescope assignment titled "Homework 4 Coding Portion".

- *OH/HWP Instructions:* There will be designated office hours for you to come to ask for conceptual and debugging help. Please visit the coding OH post for more information.

- *Academic Honesty Guideline:* We realize that code for some of the algorithms we ask you to implement may be readily available online, but we strongly encourage you to not directly copy code from these sources. Instead, try to refer to the resources mentioned in the notebook and come up with code yourself. That being said, we **do acknowledge** that there may not be many different ways to code up particular algorithms and that your solution may be similar to other solutions available online.