

ME 103 Experimentation and Measurements

Lab 3 - Vibrating Beam Sensor Calibration

Introduction and Objectives

The purpose of this lab is for you to familiarize yourself with the process of calibrating a sensor and analyzing uncertainty. The lab introduces these concepts through the means of a vibrating beam assembly, using which you will calibrate a strain gauge, linear variable differential transducer (LVDT) and accelerometer while using a linear encoder as a reference. Generally, calibrations are provided by the manufacturer when sensors are purchased off-the-shelf. However, it is crucial to be able to perform your own calibrations under specific experimental conditions. The lab also presents the opportunity for you to explore the application and benefits of frequency domain analysis. A successful Lab 3 will calibrate each sensor and utilize frequency analysis to determine the stiffness constant of the beam.

Lab Objectives:

- Understand the purpose of and be able to create and interpret calibration curves.
- Calibrate the strain gauge, accelerometer, and LVDT against the linear encoder.
- Measure the natural frequency of a vibrating beam with various end masses.

Equipment

All measurements in this lab will be performed with the NI USB-6211 DAQ Extension. Other lab-supplied materials include:

- Vibrating beam assembly
- NI USB-6211 DAQ Extension
- Analog waveform generator
- Coaxial cables
- Tape measure or ruler
- Standard masses
- Scale

Datasheets/manuals for these pieces of measurement equipment are provided in bCourses under Files > Datasheets and Manuals.

Deliverables

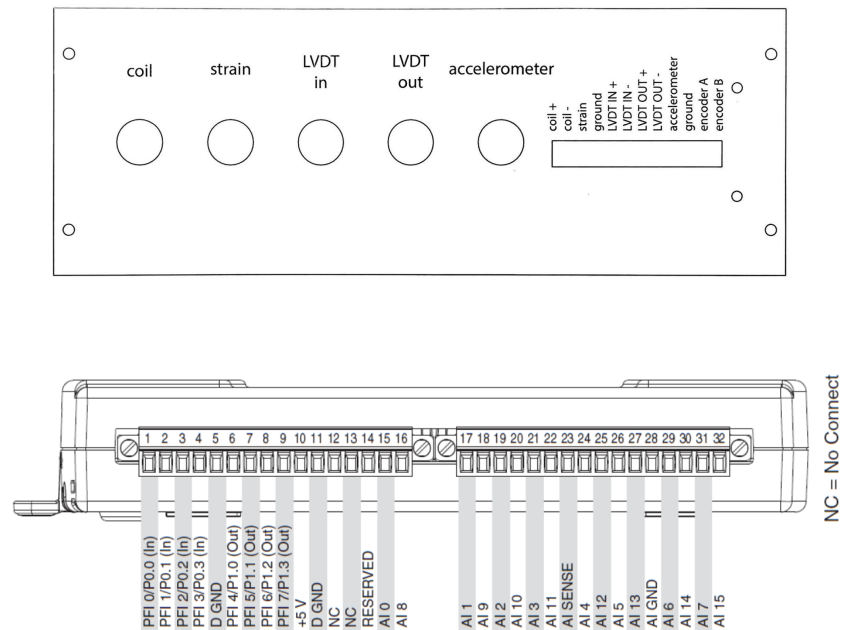
It is to *your benefit* to look at the questions *in advance* to know what you are measuring and why. With your group follow the steps below to complete the lab. You **must** typeset your answers in L^AT_EX (We recommend using Overleaf with the template provided, but you can also edit locally if you prefer). Upload a single pdf file to Gradescope per team. Everyone should be contributing equally and writing on the document equally. It is suggested to develop a shared MATLAB script containing all your (well commented) code for data analysis and plotting.

The lab is due 1 minute before your next lab section i.e the week of October 13th. For example, if you have lab on Monday at 8:00 AM, it is due the following Monday at 7:59 AM.

1 Sensor Calibration

First, we will explore the process of calibrating a sensor. To do so, we will take simultaneous measurements of various kinematic quantities of the vibrating beam using different sensors. Once the measurements are complete, we will then analyze the data to develop a relationship between the kinematic quantities and the sensor outputs (called a calibration curve).

1. You should be familiar with the working principles behind encoders, strain gauges, accelerometers, and linear variable differential transducers (LVDTs). If not, review your pre-lab.
2. Download SP26_ME103_Lab3.vi from bCourses. For this part, set the measurement time to 1 second.
3. Power the vibrating beam assembly using a 5V power supply (you can plug it into your workstation or use the DC power supply). Then connect the sensor pins from the far right side of the vibrating beam assembly to the NI USB DAQ Extension in the specified configurations. A pinout of the vibrating beam assembly and DAQ is provided below. If wiring is already connected, please check the connections.



- Connect one “ground” channel on the assembly to a DGND channel on the DAQ.
 - Connect the other “ground” channel on the assembly to the AI GND channel on the DAQ.
 - Connect the encoder A and encoder B pins to PFI0 In and PFI1 In (order doesn’t matter).
 - Connect the strain gauge output to AI1 (single-ended).
 - Connect the accelerometer output to AI2 (single-ended).
 - Connect LVDT IN +/- to AI3 (differential).
 - Connect LVDT OUT +/- to AI4 (differential).
 - Recall that “**single-ended**” means the channels share a ground and “**differential**” means that a channel’s ground is immediately adjacent to it.
4. Use the analog waveform generator to produce a sinusoidal, 4 kHz wave with a peak-to-peak amplitude of 8 V.
 5. Send the signal generated in Step 4 to the LVDT IN BNC adapter on the vibrating beam assembly.

6. Lightly flick the beam and then run the VI. You should see outputs for the encoder, strain gauge, accelerometer, and LVDT plots. Study the encoder plot (output in inches), you will use this to answer **Q1** and **Q2**.
 - You should repeat step 6 several times to observe any patterns, an even try starting the VI before flicking the beam.
 - Zoom into the LVDT plots and make note between the LVDT input and output signals.
7. Once you ensure all sensors are producing outputs, run and save **at least 10 trials** of data. This data will be used to perform calibrations.

Warning: Be sure to save the data from all trials (without known blunders) during this step. You will need to export the data to your personal device later so that you can perform analysis in MATLAB. You can transfer the data by, e.g., temporarily logging into Google Drive on the lab computer and uploading your files.

2 Natural Frequency and Flexural Rigidity

In this part, we will explore using frequency domain analysis to determine the flexural rigidity of the vibrating beam. Before performing the next steps, briefly research the natural frequency of vibrating beams.

10. Recall how the frequency resolution of a Fourier transform can be affected by measurement time. Depending on the resolution you want, adjust the measurement time accordingly (aim for a resolution of roughly 0.25 Hz or finer). The sampling frequency of the VI is 40 kHz.
11. Using the standard masses, place one additional mass on the end of the beam. Be sure to measure and record the value and uncertainty of the additional mass. There is a weighing scale in the corner of the lab near the entrance door.
12. Lightly flick the beam and run the VI. Save the data with a descriptive file name with which you can identify the added mass as you will need to analyze the data later.
13. Repeat Step 12 for 10 trials of that mass.
14. Repeat Steps 11-13 for 3 additional added masses. **You should be doing 10 trials per mass.** This means, in total, you should have **forty** different .csv files, with 10 trials each for 4 different values of added mass. With the 10 trials from the calibration step (part 1), you should have **50** total .csv files. *Please name them carefully!* Fill in the table in **Q11** as you go.
15. Measure the length of the beam using the ruler and note the uncertainty.
16. **This is not a step.** Answer the following questions **Q12, Q13, Q14, Q15, Q16**. Assume the vibrating beam is modeled as a simple cantilever beam where the beam body weight is negligible and the head weight acts as a point force at the free end. Answer the questions for this part.

Note: The equation for natural frequency for a cantilever beam with end mass concentration is given as

$$f_{\text{nat}} = \frac{1}{2\pi} \sqrt{\frac{3EI}{ML^3}} \implies EI = \frac{(2\pi f_{\text{nat}})^2 ML^3}{3}$$

Questions

Section 1 Questions

1. Study the encoder plot. Note the output is in inches. How does the encoder define a “zero” measurement?
2. Compare the LVDT input and output signals. Describe how the differences between the signals correlate to the physical behavior of the beam.
3. Recall what kinematic quantities are measured by the strain gauge and encoder. Are they the same? Predict the type of relationship that exists between the strain gauge output voltage and encoder measurement (i.e. linear, quadratic, etc.). Briefly explain your prediction.
4. Using MATLAB or another data processing software, calibrate the strain gauge using the encoder data as a reference, using the questions below as a guide. Download `sg_calibration_skeleton.m` from bCourses.
5. Using what you learned in Question 1, adjust/correct the encoder measurements from all trials so that the equilibrium of the beam corresponds to zero displacement. (Hint: how can you determine the “equilibrium” position of any oscillatory data?)
6. Create a graph that allows you to visualize the relationship you predicted in Question 3. Plot the data from all trials. Think about which quantity should be plotted on which axis (if we want a kinematic quantity as a function of the strain gauge voltage, should the voltage be plotted on the horizontal or vertical axis?). You may reference `preprocessing_skeleton.m`.
7. Repeat **Q4**, **Q5**, **Q6** with the accelerometer and LVDT. You can use `accel_calibration_skeleton.m` and `lvdt_calibration_skeleton.m`. Note that each sensor may require additional pre-processing.
8. Generate a best-fit curve for the graph you produced in Question 6 and plot it on the same graph. Consider carefully the most appropriate order for the polynomial you will fit. MATLAB's `polyfit()` and `polyval()` may be helpful. Consider carefully the most appropriate order for the polynomial you will fit (bearing in mind the functional relationship you predicted in Question 3 and the need to avoid ‘overfitting’). You may reference `sg_calibration_skeleton.m`.
9. Obviously the data do not all lie on the best-fit curve. What might be some reasons for this? Generate a 95% confidence interval for your best-fit-curve by determining the 95% confidence interval for the constant term in your curve. Again MATLAB's `polyval()` may be helpful. Plot the upper and lower bounds of your best-fit-curve confidence interval on the same graph as Questions 5 and 6. You may reference `sg_calibration_skeleton.m`.
10. Verify that your graph axes are labeled with units and the graph itself is titled. You may reference `sg_calibration_skeleton.m`.
11. Report (symbolically/mathematically) your best-fit-curve (with units) with 95% uncertainty. This equation is the calibration curve for the strain gauge. You should be able to plug in to your curve a strain gauge output voltage and obtain a kinematic quantity with 95% uncertainty. You may reference `sg_calibration_skeleton.m`.
12. Ensure that for each sensor (strain gauge, accelerometer, LVDT), you have reported a correctly formatted graph that includes experimental data, a best-fit-curve, and a 95% confidence interval. Additionally, ensure that for each sensor, you have mathematically reported your calibration curve equation with units and uncertainty.

Section 2 Questions

11. Complete the table below with your data.

Added Mass	Total Mass	Mass Uncertainty	Natural Frequency	Frequency Uncertainty
0 kg	0.0571 kg			

12. Would you expect the flexural rigidity to be dependent on the added mass? Why or why not?
13. For each mass, calculate the flexural rigidity and its uncertainty.
14. Compare your calculations to your answer to Question 12. Was your prediction correct in context of the uncertainties? If not, what might be some reasons for the difference?
15. Consider what uncertainties you propagated to calculate the uncertainty of the flexural rigidity. What was a benefit of using frequency domain analysis in this particular experiment?
16. Perform a Fourier analysis on any **one** of the sensors, and use it to determine the natural frequency of the beam for each of the 5 mass conditions (no added mass + 4 additional masses). The mass of the beam head without any added masses is 57.1 g. (Hint: MATLAB has a built-in `fft()` function). Please submit a screenshot of each `fft` plot (5 total).

Section 3 Questions

In this section of the lab, you will further characterize the vibrating beam by determining its damping ratio. From an elementary physics or dynamics class i.e. ME104, recall that free oscillations can be characterized by their natural frequency and a dimensionless damping ratio:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0$$

Solving this differential equation for underdamped systems (ζ between 0 and 1) reveals that they oscillate with an exponentially decreasing amplitude, with position given by:

$$x(t) = Ae^{-\omega_n\zeta t} \cos(t\omega_n\sqrt{1-\zeta^2} + \varphi)$$

where A is a constant and φ is a phase shift. The questions in Section 3 will guide you through determining the damping ratio.

17. A useful quantity when analyzing underdamped systems is the logarithmic decrement δ , which is the natural logarithm of the ratio of successive peaks, i.e. $\delta = \ln(\frac{A_0}{A_1})$. Use the 2nd equation above in this section to obtain an expression for the logarithmic decrement in terms of the damping ratio.
- **Note:** We do not simply define the ratio of successive peaks because the amplitude decays exponentially and therefore the natural log of the ratio is a constant!
 - *Hint:* Using a phase shift of 0 for simplicity, think about what values of t give a maximum value for the cosine term.
18. Choose one of the sensors you calibrated in Part 1 (strain gauge, accelerometer, or LVDT). Write a MATLAB script to extract several peaks of the readings from that sensor from one trial of the data from section 2. Calculate the logarithmic decrement between each pair of successive peaks and use their average value to estimate the damping ratio for that trial
- *Hint 1:* Your solution to EQ1 will be useful here.

- Hint 2: There are several ways to extract peaks. You performed a frequency analysis on your data in Part 2; you can take advantage of this and perform further frequency analysis to determine the peaks. Alternatively, cleverly using MATLAB's `findpeaks()` function should allow you to extract the peaks.
19. (a) Using uncertainty propagation, calculate the uncertainty in the damping ratio.
- (b) Inspect the uncertainties that you propagated. Are there any similarities or differences compared to the uncertainty analysis you performed in Part 2?
- (c) Would you expect the uncertainty in the damping ratio to increase or decrease as you increase the number of peaks you analyze? Can you demonstrate your reasoning mathematically?