HW2

September 24, 2026 at 10:20am

Reminder: Submit your assignment on Gradescope by the due date. Submissions must be typeset. Each page should include work for only one problem (i.e., make a new page/new pages for each problem). See the course syllabus for the late policy.

While collaboration is encouraged, please remember not to take away notes from any labs or collaboration sessions. Your work should be your own. Use of other third-party resources is strictly forbidden.

Please monitor Ed discussion, as we will post clarifications of questions there.

Problem 1

(a) Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and non-accept states in M doesn't necessarily yield a new NFA that recognizes the complement of C.

(b) Is the class of languages recognized by NFAs closed under complement? Explain your answer.

Problem 2

Let Σ and Δ be finite alphabets of symbols. A *homomorphism* is a function $h : \Sigma^* \to \Delta^*$ defined as follows:

(a) $h(\epsilon) = \epsilon$;

(b) For any $a \in \Sigma$, $h(a) \in \Delta^*$;

(c) For $a \in \Sigma^*$ such that $a = a_1 a_2 \dots a_n$ with $n \geq 2$,

$$h(a) = h(a_1)h(a_2)\dots h(a_n).$$

That is, a homomorphism is a function from strings to strings that "respects" concatenation: for any $x, y \in \Sigma^*$, $h(xy) = h(x)h(y)$.

Example: $h : \{0, 1\} \to \{a, b\}^*$ where $h(0) = ab$ and $h(1) = ba$. Then $h(0011) = ababbaba$.

Given a homomorphism $h : \Sigma^* \to \Delta^*$ and a language $L \subseteq \Sigma^*$, we define the homomorphism of a language L as $h(L) = \{h(w) \mid w \in L\} \subseteq \Delta^*$.

Prove that the set of regular languages is *closed under homomorphism*, that is, for any given regular language L, $h(L)$ is a regular language.

Problem 3

We have already seen that the language

$$L = \left\{ w \;\middle|\; \begin{array}{c} w \text{ contains an equal number of} \\ \text{occurrences of the substrings 01 and 10} \end{array} \right\}.$$

is regular. Now let

$$L' = \left\{ w \;\middle|\; \begin{array}{c} w \text{ contains an equal number of} \\ \text{occurrences of the substrings 010 and 101} \end{array} \right\}.$$

Show that L' is not regular.

Hint: if you're struggling to find a good string, consider one whose first segment contains only one of the substrings 010 or 101.

Problem 4

Consider the language over the alphabet $\Sigma = \{x, y, z\}$:

(a) Let

$$L_1 = \left\{ x^k w \;\middle|\; \begin{array}{c} k \neq 1 \text{ and } w \in \Sigma^* \text{ and the} \\ \text{first symbol of } w \text{ is not an `}x\text{'} \end{array} \right\}.$$

Prove that L_1 is regular. Showing a correct DFA/NFA/RegEx is sufficient.

(b) Let

$$L_2 = \{xy^n z^n \mid n \geq 0\}.$$

Prove that L_2 is not regular.

(c) Prove that $L = L_1 \cup L_2$ satisfies the pumping lemma property.

(d) Is L regular? Provide a proof for your answer.

Hint: it might help you to show that regular languages are closed under complement and/or intersection...