ME 103 — Experimentation and Measurements — Spring 2026 — George Anwar

Pre-Lab VB

Due: Tuesday, February 24th at 7:59 am

- This pre-lab will cover the vibrating beam apparatus and related measurement techniques and sensors.
- In all of the questions, **show your work**, not just the final answer. Unless we explicitly state otherwise, you may expect full credit only if you explain your work succinctly, but clearly and convincingly. For coding questions, attach a screenshot of your code and output.
- Present your answers with **a suitable number of significant figures** for each question. Show your work, including a mathematical formula or the MATLAB or Python code you wrote, before reaching the result. You may need to install the Statistics Toolbox if using MATLAB.
- Throughout this assignment, neglect systematic (bias) errors. Also, assume a normal distribution for the underlying distribution (population) if necessary.
- If you have a confirmed disability that precludes you from complying fully with these instructions or with any other parameter associated with this problem set, please alert us immediately about reasonable accommodations afforded to you by the DSP Office on campus.

Deliverables

Submit a PDF of your pre-lab to the **Gradescope assignment** entitled "{Your Name} Pre-Lab VB". **You may** typeset your homework in LaTeX (submit PDF format, not .doc/.docx format) . Mac Preview, PDF Expert, and FoxIt PDF Reader, among others, have tools to let you sign a PDF file. We want to make *extra clear* the consequences of cheating. You may work with your lab group but make sure everyone understands the lab!

Introduction

The purpose of this pre-lab is for you to familiarize yourself with the process of calibrating a sensor and analyzing uncertainty. The lab introduces these concepts though the means of a vibrating beam assembly, using which you will calibrate a strain gauge, linear variable differential transducer (LVDT) and accelerometer while using a linear encoder as a reference. Generally, calibrations are provided by the manufacturer when sensors are purchased off-the-shelf. However, it is crucial to be able to perform your own calibrations under specific experimental conditions. The lab also presents the opportunity for you to explore the application and benefits of frequency domain analysis. Before coming to lab, your team must be able to:

Pre-lab Objectives:

- Explain what each sensor in the vibrating beam apparatus measures and what signal type it outputs (analog vs digital).
- Understand the **physical dynamics** of the vibrating beam system.
- Sketch and derive the free-oscillation equation starting from a free-body diagram (FBD) and an equivalent 1-DOF model.
- Propose an experimental plan to estimate ω_n and ζ from measured data (you will confirm implementation details by reading the lab handout + VI during lab)
- Anticipate what pre-processing and filtering steps will be required for each sensor channel.
- How signals move through the data acquisition system in LabVIEW.

The lab objective is to calibrate sensors (strain gauge, accelerometer, LVDT) against the encoder and use frequency analysis to extract dynamic parameters.

1 Apparatus Overview and Sensor Operation

This section introduces the mechanical system and sensing components used in the experiment. Before collecting data, it is important to understand how the **beam moves, what each sensor measures, and how those measurements relate to the system's dynamics**. The pinout for the vibrating beam apparatus is

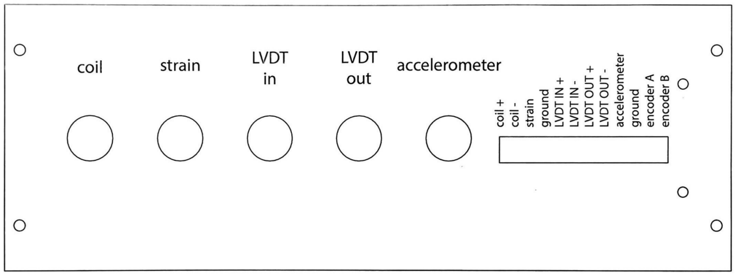


Figure 1: Pinout for Vibrating beam apparatus

Each sensor observes a different physical quantity, i.e. displacement, strain, or acceleration, and each measurement passes through electronics and data acquisition hardware before being analyzed. Developing a clear understanding of the apparatus and sensor operation will help you interpret signals correctly, diagnose experimental issues, and design appropriate methods for estimating system parameters.

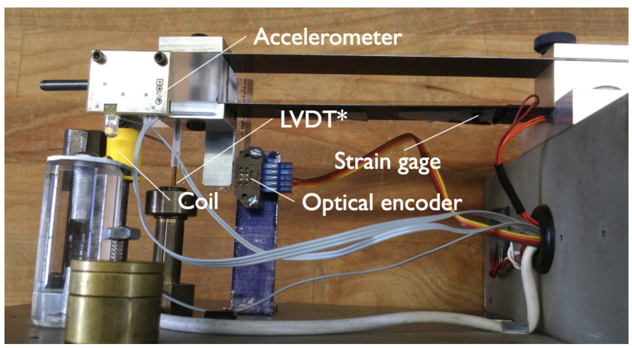


Figure 2: Vibrating beam apparatus with coil actuator and sensors

1.1 Importance of Grounding

In measurement systems, grounding provides a common electrical reference and a safe path for unwanted currents, helping ensure that sensor signals accurately represent physical motion rather than electrical artifacts. However, improper grounding can introduce unwanted DC offsets or noise into measurements, corrupting data collected from sensitive sensors such as strain gauges, accelerometers, and LVDTs.

For this reason, all circuit blocks and sensors in the experiment should share a common ground reference, ideally arranged in a star-ground configuration, where all ground connections meet at a single point rather than forming loops. Ground loops should be avoided because time-varying magnetic fields—such as those produced by nearby power lines or laboratory equipment operating at 60 Hz—can induce voltages in any closed conducting loop according to **Faraday's Law**,

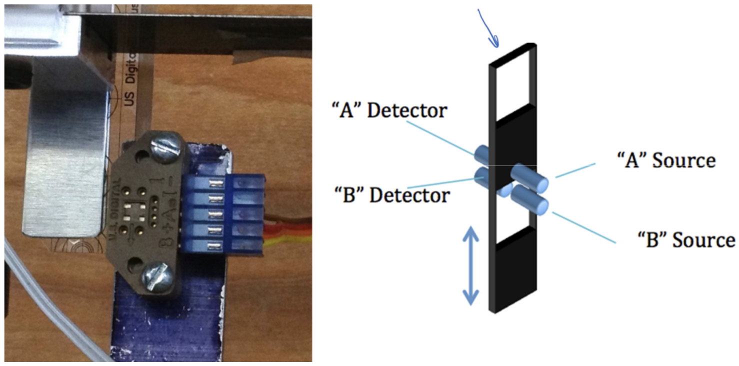
$$\Delta V = -\frac{d\Phi_B(t)}{dt}, \qquad \Phi_B(t) = \iint_{\Sigma} B(t)\, dA,$$

where Σ is the area enclosed by the conducting loop and Φ_B is the magnetic flux passing through that area. A larger loop encloses more area and therefore intercepts more magnetic flux, leading to a larger induced voltage. If this loop is part of the ground wiring, different points along the ground conductor may sit at slightly different voltages, introducing noise that can become comparable to the small sensor signals being measured.

1.2 Sensors in the Apparatus

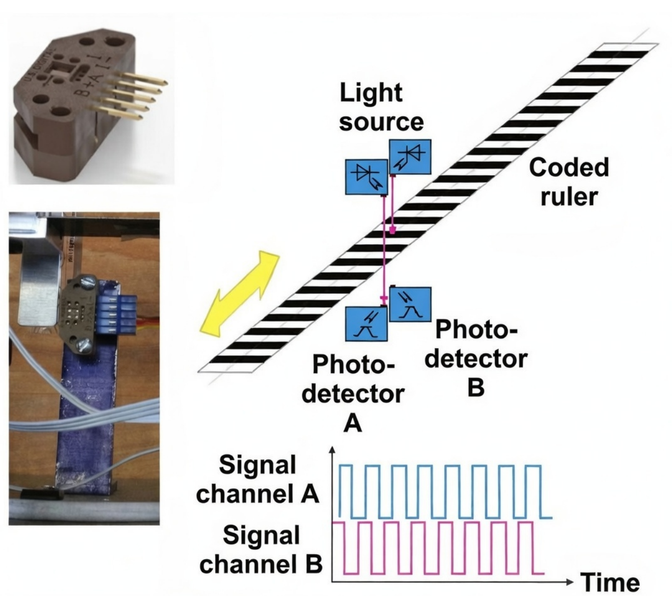
The apparatus has 5 types of sensors: and optical encoder with 4x quadrature and no index; an LVDT or linear variable differential transformer; accelerometer that measures acceleration; strain gauge that acts as a variable resistor; and a voice coil that utilizes Faraday's law. In this section we will walk through each sensor in detail.

1.2.1 Optical Encoder



We now move into the mechanical sensing components used in the experiment, focusing first on the optical linear encoder, which provides an accurate and intuitive method for measuring **relative position**. The encoder operates using a thin strip attached to the moving beam that contains alternating transparent and opaque markings. As the beam oscillates, this patterned strip moves through a sensing module containing a light source (typically an LED) and photodetectors positioned on the opposite side.

When a transparent section passes between the light source and detector, light reaches the detector and produces a high signal. When an opaque section blocks the light, the detector output drops. As motion continues, this alternating pattern produces a sequence of pulses corresponding to displacement. By counting these pulses, the system determines how far the beam has moved. Because the beam oscillates rapidly, the markings move quickly past the detectors, generating a high-rate pulse signal that is converted by the data acquisition system into a continuous displacement measurement.



Why are two sensors needed in an optical encoder? This is required to utilise a method called quadrature encoding, in which two photodetectors are positioned so that their signals are offset by one quarter of a cycle. If only a single light source and detector were used, the output would simply oscillate as the patterned strip moved, allowing measurement of motion frequency and therefore speed, but not direction.

With two detector channels offset in quadrature, however, the sequence in which signal transitions occur reveals the direction of motion. For example, if a rising edge appears on channel A slightly before the corresponding transition on channel B, motion is occurring in one direction; reversing the direction reverses the order of these transitions. Internal electronics process these signals, count pulses, and incorporate direction information to produce a continuously updated displacement measurement.

Because the encoder strip consists only of repeating stripes, there is no inherent absolute zero position. When the system powers on, the position is effectively set relative to the current location, so the apparatus must either begin from a known reference position or be referenced during operation.

For this encoder, the coded ruler has 250 lines per inch, meaning there are 250 repeating stripe cycles per inch of motion. Each cycle produces four detectable signal edges—two from each channel—which can all be counted. Using this information, the theoretical resolution can be estimated as

$$\frac{25400\,\mu\text{m/in}}{250\text{ cycles/in} \times 4\text{ edges/cycle}} = 25\,\mu\text{m}$$

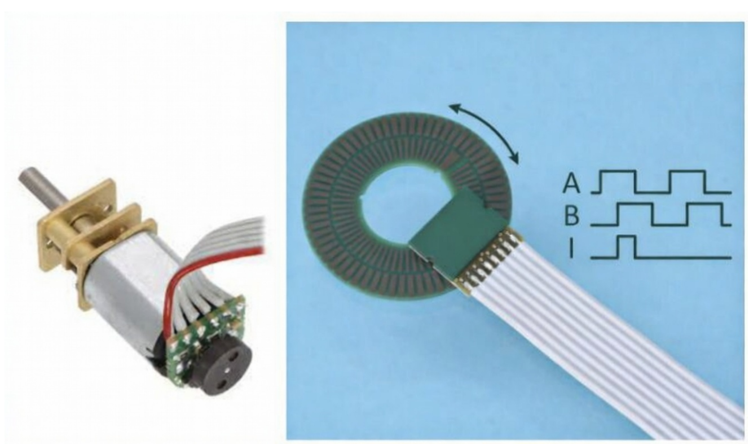
or approximately one thousandth of an inch per detectable step.

1.2.2 Encoders General

An encoder is a device that converts motion—either linear or rotational—into a digital signal representing position. In practice, motion of a patterned or graduated scale is sensed and converted into electrical signals that encode position information. The sensing method does not have to rely solely on optical transmission through a transparent strip; it may also use optical reflection, electromagnetic sensing, or even mechanical contact. For example, some small motor encoders use embedded magnets and Hall-effect sensors to detect changing magnetic fields rather than light. Mechanical contact encoders also exist but are less common today due to wear and reliability concerns.

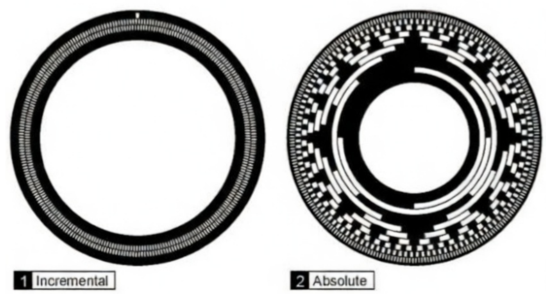
Encoders can measure either rotational motion (rotary encoders) or linear motion (linear encoders), both of which follow similar encoding principles.

One major classification is between incremental (relative) and absolute encoders. Incremental encoders, such as the one used in the Lab 3 apparatus, consist of repeated markings or stripes. As motion occurs, pulses are generated and counted to determine how much movement has occurred relative to a starting point. Direction information is typically obtained using the quadrature method, which uses two offset channels. Some incremental encoders also include an index channel, producing a single pulse per cycle to provide a reference location. However, if pulses are missed due to noise or rapid motion, position errors can accumulate until the index position is encountered again.



An alternative approach is absolute encoding, in which every possible position corresponds to a unique digital code. This avoids cumulative errors because the encoder always reports the true position directly.

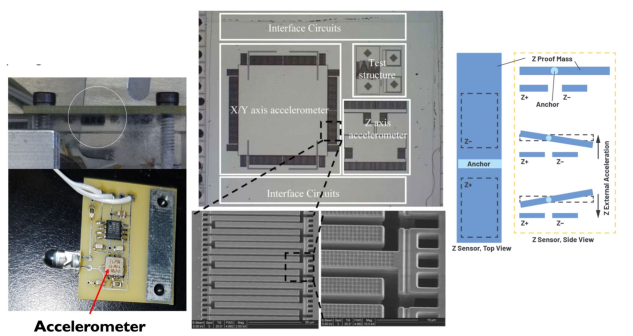
Absolute encoders require multiple sensing channels corresponding to binary digits. To reduce errors during transitions between positions, many systems use Gray code, in which only one bit changes between adjacent positions. This prevents transient misreadings that could occur if multiple bits changed simultaneously, ensuring more reliable position detection.



	Gray code			
	4	3	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	1
3	0	0	1	0
4	0	1	1	0
5	0	1	1	1
6	0	1	0	1
7	0	1	0	0
8	1	1	0	0
9	1	1	0	1

Incremental encoders are simpler and cheaper, while absolute encoders provide higher reliability in applications where position must be known immediately after power-up or where missing counts cannot be tolerated.

1.2.3 Accelerometer



The Lab 3 apparatus also includes an accelerometer, a sensor that measures acceleration and is now found in nearly every mobile device. Modern accelerometers are remarkable examples of micro-engineering, fabricated using micro-machining techniques on silicon chips. Inside the device, a small movable structure called a proof mass is formed from silicon and suspended by microscopic spring-like structures.

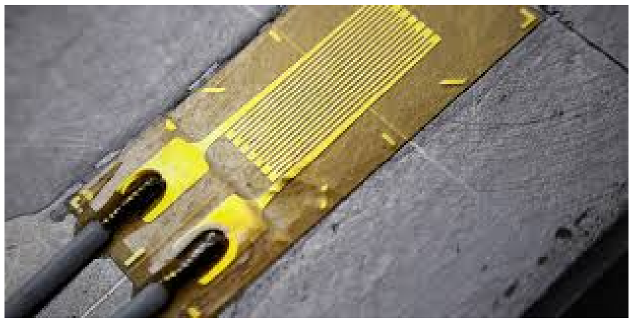
When the sensor experiences acceleration, the inertia of the proof mass causes it to resist motion relative to the surrounding structure. As a result, the supporting springs deform slightly, producing a displacement proportional to the applied acceleration. This motion is extremely small—often only fractions of a micron—but can still be detected electrically.

Detection is accomplished using arrays of interlocking microscopic “fingers” that form capacitors. Some fingers are attached to the moving proof mass, while others are fixed to the sensor body. As the proof mass shifts under acceleration, the spacing between these fingers changes, slightly altering the capacitance. By driving the structure electrically and measuring changes in voltage or charge associated with these capacitors, the electronics determine how much the mass has moved and therefore infer the applied acceleration.

Acceleration in multiple directions can be measured by arranging sensing fingers in different orientations. Motion within the plane of the chip is measured using lateral finger motion, while acceleration perpendicular to the chip surface (the Z-axis) is often measured using a slightly different structure in which the proof mass tilts or moves vertically, producing changes in capacitance between electrodes above and below the mass.

This technology allows compact sensors to measure motion in three dimensions, which is why accelerometers are now standard components in smartphones, robotics systems, and the Lab 3 experimental apparatus.

1.2.4 Strain Gauge



We now turn to strain gauges, one of the most widely used measurement tools in mechanical engineering. Strain gauges are commonly used in material testing, structural monitoring, and experimental mechanics to measure deformation in components and structures over time.

A strain gauge typically consists of a thin metallic resistive track arranged in a serpentine pattern and mounted on a thin insulating backing. When the gauge is firmly bonded to the surface of a structure, any deformation of that structure produces a corresponding deformation in the gauge. As the structure stretches or compresses, the resistive track changes length and cross-sectional area, causing a small but measurable change in electrical resistance.

The serpentine geometry allows a long conductor length to be packed into a small area, increasing sensitivity while ensuring that the gauge primarily responds to strain along a specific direction. As a result, strain gauges provide directional strain measurements, meaning the orientation of the gauge on the structure determines which strain component is measured. To obtain strain information in multiple directions, multiple gauges—often arranged in configurations known as strain rosettes—are used.

Transduction from resistance to voltage for strain gauges

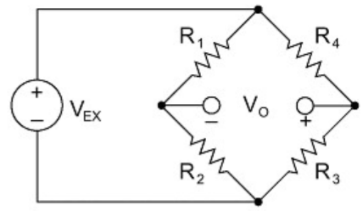
Strain gauges are manufactured in large quantities and therefore come with a specified gauge factor, defined as

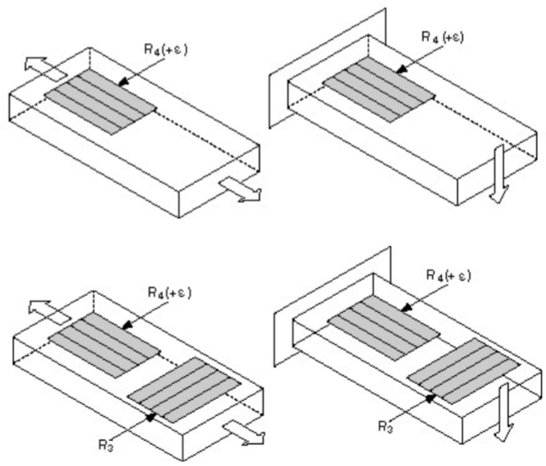
$$G = \frac{\Delta R/R}{\Delta L/L} = \frac{\Delta R/R}{\epsilon}$$

which relates the relative resistance change to mechanical strain. Because the strains encountered in most engineering applications are very small—typically far below one percent—the resulting resistance changes are also extremely small. Therefore, a circuit is needed to convert these tiny resistance changes into a measurable voltage signal that can be amplified and digitized.

Typical strain gauges have resistances ranging from roughly 120Ω to $1\text{k}\Omega$. Higher resistance gauges generally reduce current flow and therefore reduce self-heating, which can otherwise introduce measurement errors, although they may also be somewhat more susceptible to electrical noise depending on circuit design.

To measure these small resistance changes effectively, strain gauges are commonly used within a Wheatstone bridge, a circuit consisting of four resistors arranged in a diamond configuration. In the simplest case—known as a quarter-bridge configuration—one resistor is replaced by the strain gauge while the remaining three are fixed resistors. When no strain is present, all four resistances are equal, and the bridge output voltage is zero because both sides of the bridge form identical voltage dividers. When strain slightly changes the resistance of the gauge, the balance of the bridge is disturbed, producing a small differential output voltage that can be amplified and measured accurately. This differential measurement suppresses large background voltages and improves sensitivity.





Strain gauges can be used to measure both axial tension and bending. For example, when a beam bends, one surface experiences tension while the opposite surface experiences compression. A gauge bonded to the surface undergoing tension will increase in resistance, producing a corresponding change in bridge output voltage.

However, strain gauges are also sensitive to temperature changes, which can alter resistance even when no mechanical strain is present. To compensate for this, a second gauge—called a dummy or reference gauge—can be placed near the primary gauge so that it experiences the same temperature changes but is not mechanically strained. Incorporating this reference gauge into the bridge allows temperature effects to cancel out, leaving only resistance changes due to actual strain. This approach improves measurement reliability by removing influences unrelated to the mechanical quantity being measured.

Half-bridge Configurations

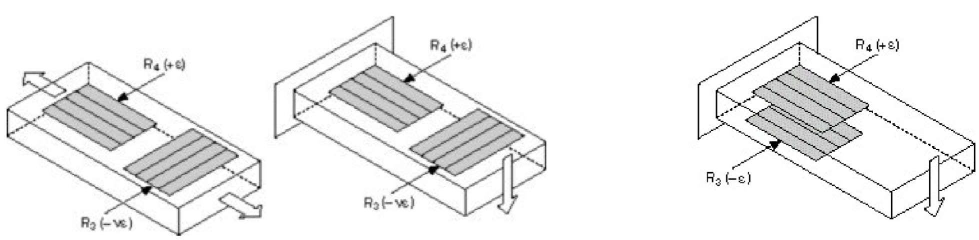


Figure 3: From left two to right: Type I,II

If higher measurement sensitivity is required, strain gauges can be arranged in a half-bridge configuration, where two of the four resistors in the Wheatstone bridge are active strain gauges bonded to the structure. In this arrangement, both gauges experience strain, allowing their resistance changes to reinforce each other and produce a larger output voltage change compared with the quarter-bridge case.

In a Type I half-bridge configuration, one gauge is aligned with the primary loading direction and experiences an increase in resistance when the structure is stretched. A second gauge is placed orthogonally, where it responds primarily to transverse strain produced by the Poisson effect—the tendency of materials to contract in directions perpendicular to applied tension. As a result, when one gauge's resistance increases, the other decreases. Because these gauges occupy adjacent arms of the Wheatstone bridge, their resistance changes compound, increasing measurement sensitivity. This configuration also provides partial temperature compensation, since both gauges experience similar environmental conditions.

Half-bridge configurations are also useful for measuring bending loads. In bending, the top surface of a beam is placed in tension while the bottom surface experiences compression. In a Type II configuration, one gauge is mounted on the tensile surface while the other is placed on the compressive surface. During bending, one gauge's resistance increases while the other decreases, again amplifying the bridge output signal.

An important advantage of these configurations is the ability to distinguish between different loading types. For example, in a Type II configuration, if the structure experiences pure axial loading rather than bending, both gauges undergo similar resistance changes, leaving the bridge output largely unchanged. By combining Type I and Type II arrangements, it becomes possible to separate axial and bending strain contributions in a structure, providing more detailed information about structural loading conditions.

Full-bridge Configurations

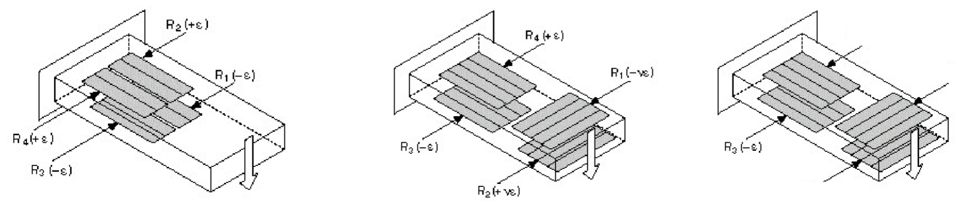


Figure 4: From left to right: Type I, II, III

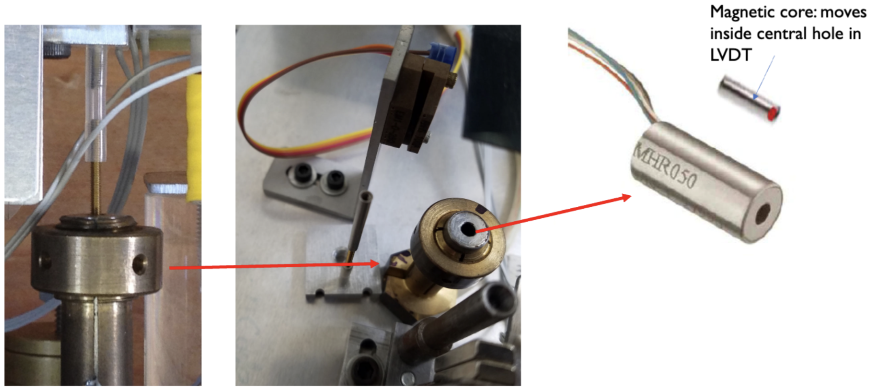
The full-bridge configuration represents the most sensitive practical implementation of strain gauge measurements, in which all four resistors in the Wheatstone bridge are active strain gauges bonded to the structure. By arranging gauges so that some experience tensile strain while others experience compressive strain, the resulting resistance changes reinforce each other within the bridge, maximizing the differential output voltage.

Under bending loads, the top surface of a beam typically undergoes tension while the bottom surface experiences compression. In a typical full-bridge bending configuration, gauges placed on the tensile surface increase in resistance, while gauges on the compressive surface decrease in resistance. These resistance changes occur in opposite arms of the bridge, causing voltage changes in opposite directions on each side of the bridge and producing a differential output that can be approximately four times larger than that obtained from a quarter-bridge arrangement. This significantly improves sensitivity without requiring additional signal amplification.

Several full-bridge layouts are possible depending on which strain components are of interest. In some configurations, gauges are oriented orthogonally so that both primary strain and transverse strain arising from the Poisson effect are measured. This arrangement can help compensate for uncertainty in Poisson's ratio when converting measured strain to stress or material properties.

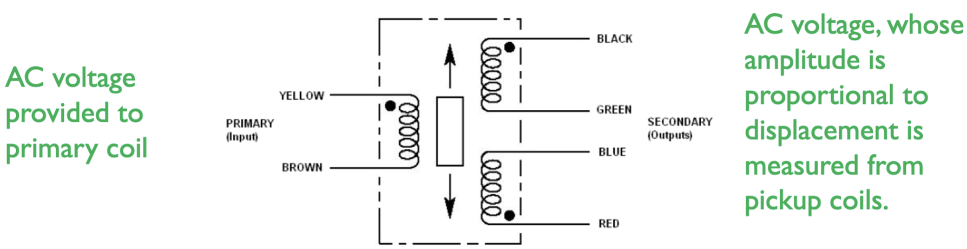
Other configurations place gauges to isolate axial strain only, arranging gauges so that resistance increases due to axial extension and decreases due to transverse contraction combine to maximize sensitivity while canceling unwanted bending effects. By carefully choosing gauge placement and orientation, full-bridge configurations allow engineers to selectively measure bending, axial loading, or combinations of strain components while simultaneously improving measurement sensitivity and environmental compensation.

1.2.5 Linear Variable Differential Transformer (LVDT)



A Linear Variable Differential Transformer (LVDT) is a widely used displacement sensor in mechanical engineering and serves as a workhorse measurement device in many experimental setups, including the Lab 3 apparatus. LVDTs are commonly mounted on tensile and compression testing machines to provide accurate measurements of specimen deformation, often offering more localized and precise displacement measurements than machine-frame encoders, which may be affected by structural compliance.

An LVDT operates as a type of differential electrical transformer used to measure linear displacement. The sensor consists of three coils arranged along a common axis: a central primary coil and two secondary coils placed symmetrically on either side. The object whose displacement is being measured is attached to a movable rod carrying a ferromagnetic core, which slides within the coils.



To operate, an AC excitation voltage, typically sinusoidal, is applied to the primary coil. This alternating current produces an oscillating magnetic field surrounding the coil. The ferromagnetic core concentrates and channels this magnetic field, increasing the magnetic coupling between the primary coil and whichever secondary coil the core overlaps more strongly.

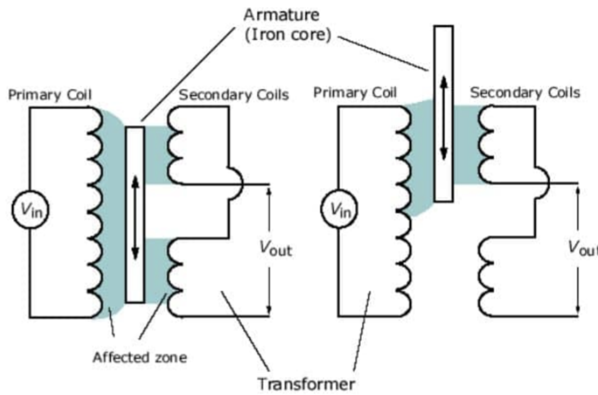
As the core moves, the magnetic coupling to the two secondary coils changes. When the core is centered, both secondary coils receive approximately equal induced voltages, and because they are wired in opposition, their outputs cancel, producing nearly zero output voltage—this position is called the null position. When the core moves toward one secondary coil, the induced voltage in that coil increases while the voltage in the opposite coil decreases. The output signal is taken as the difference between these two voltages, producing an AC output whose amplitude is proportional to displacement and whose phase indicates direction.

Thus, displacement is determined by measuring the amplitude and sign of this differential output signal. The sensitivity of the LVDT is typically specified as output voltage amplitude per unit input excitation voltage per unit displacement. Sensitivity can be increased by raising the excitation voltage, provided the coil current remains within safe limits.

Because the device relies only on electromagnetic coupling and involves no mechanical contact between sensing components, LVDTs offer high precision, durability, and repeatability, making them valuable tools in precision displacement measurement.

Magnetic Flux Coupling Depends on Core Displacement

To understand how an LVDT converts displacement into voltage, we need to examine how magnetic flux is generated and manipulated inside the device. When the ferromagnetic core is centered, magnetic flux from the primary coil couples approximately equally into both secondary coils. When the core moves off-center, flux preferentially couples into one coil more than the other, producing a differential output voltage proportional to displacement.



Two key electromagnetic quantities appear: magnetic field intensity, H and the magnetic flux density B. These are related through the magnetic permeability $B = \mu H = \mu_r \mu_0 H$ where μ_r is the relative permeability of the material and μ_0 is the permeability of free space. For iron, $\mu_r \sim 1000$, whereas for air $\mu_r \sim 1$. This large difference explains why the iron core strongly concentrates magnetic flux within the LVDT. The oscillating current applied to the primary coil produces the magnetic field according to Ampère's law, expressed:

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu I$$

This states that the line integral of the magnetic field around a closed path equals the permeability times the current passing through the loop. For example, around a straight wire carrying current I, the magnetic field circulates around the conductor. For a circular path of radius r,

$$B(2\pi r) = \mu I \implies B = \frac{\mu I}{2\pi R}$$

The direction follows the right-hand rule: thumb along current, fingers show magnetic field circulation. In an LVDT, this oscillating current produces an oscillating magnetic field concentrated by the iron core. This is an example of why differential measurements help: External magnetic disturbances in the lab tend to affect both secondary coils similarly. However, motion of the core changes coupling differently in each coil. By measuring the difference between the secondary voltages, common disturbances cancel while displacement-dependent signals remain.

Why is the design of the LVDT coaxial?

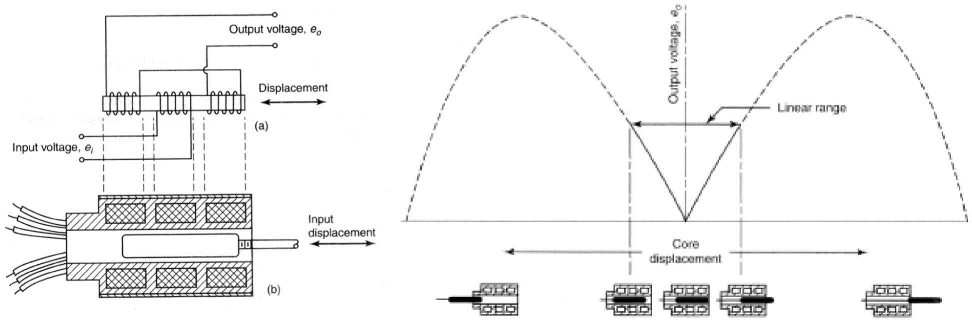
An LVDT uses a coaxial arrangement where the primary and secondary coils share a common axis and the movable iron core travels through their centers. This geometry maximizes sensitivity and improves linearity of the output signal with respect to displacement.

For a long solenoid, magnetic flux is concentrated inside the coil and is very weak outside. Placing the core along the coil axis ensures it interacts with the region of strongest magnetic field, improving coupling to the secondary coils.

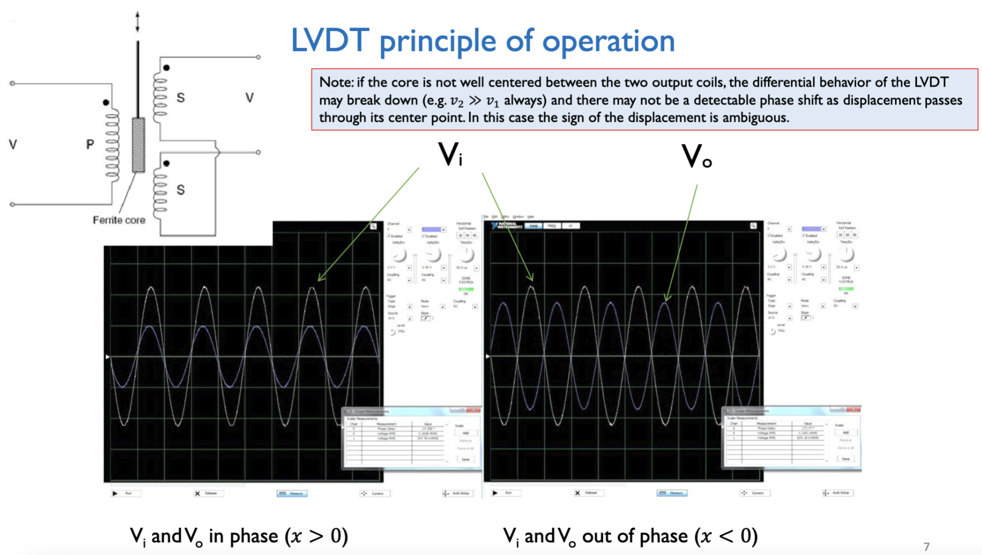
The iron core, with high relative permeability, further concentrates magnetic flux, so moving the core shifts flux preferentially toward one secondary coil or the other. Measuring the voltage difference between the coils produces a displacement signal that is both stronger and more linear than would be possible with a single coil or non-coaxial geometry.

LVDT principle of operation

How do we know whether we are left or right of center? The plot below shows the magnitude of the LVDT output voltage, which is symmetric about the center position. Therefore, magnitude alone cannot tell us the direction of displacement.



The direction is determined from the phase of the output signal relative to the input excitation. When the core moves to one side, the output voltage is in phase with the excitation; when it moves to the other side, the output becomes 180° out of phase. By comparing output phase with the input signal, we can determine whether the core is left or right of center.



Because the secondary coils are connected so the output is ($V_{\text{out}} = V_2 - V_1$), the output voltage retains sign information, not just magnitude.

In practice, this sign appears as a phase difference between the excitation voltage and the output signal. When displacement (x) is positive, the output voltage is approximately in phase with the input. When (x) is negative, the polarity reverses and the output becomes 180° out of phase with the excitation.

This is why both the input excitation signal and the LVDT output are recorded: comparing their phases allows us to determine not only the magnitude of displacement but also whether the core is to the left or right of center.

1.2.6 Coil Electromagnetic Actuator

A current flowing through a coil creates a magnetic field. When placed near a magnet or magnetic material attached to the beam, this produces a force. Thus, the coil acts as a force input to the beam.

Section 1 Questions

1. Why is proper grounding important in measurement systems, and what problem do ground loops introduce?
2. Fill in what each element measures and what quantities can be computed from it.

Element	Primary physical quantity	What you can compute from it
Encoder		
Strain gauge		
Accelerometer		
LVDT		
Coil		

3. What does the **encoder** do? How will it help with our experiment? Roughly describe how does it work?
4. Why are two channels used in quadrature encoding instead of a single detector?
5. Why does an incremental encoder not provide an absolute position after power-up?
6. How does a strain gauge convert mechanical deformation into an electrical signal?
7. Why is a Wheatstone bridge used with strain gauges?
8. What does the **accelerometer** do? How will it help with our experiment? Roughly describe how does it work?
9. What physical quantity does an LVDT measure, and how does it determine direction of motion?
10. How does the coil act as an actuator in the apparatus?

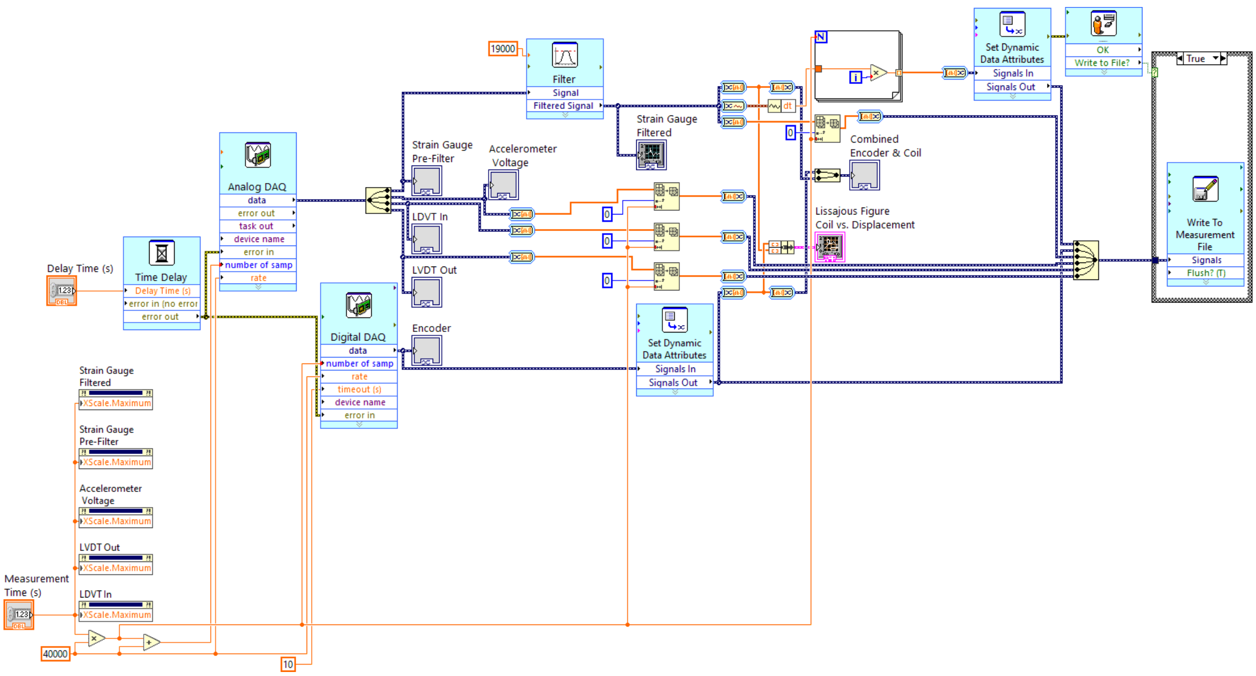
2 Data Acquisition and Signal Flow

We know that most of you have not yet looked at the LabVIEW VI in detail, and analyzing LabVIEW block diagrams is not the goal of this class. However, as experimental engineers, it is important that you understand what is happening to your data while measurements are being taken. You are not just reading numbers from a screen, you are observing signals that have traveled through sensors, electronics, data, acquisition hardware, and software processing before appearing in plots and files.

This section provides a conceptual map of how your measurements flow through the system so that when you collect data, you understand what the instruments are actually doing. During Lab 3, signals move through the following chain:

$$\text{Mechanical Motion} \rightarrow \text{Signal Conditioning} \rightarrow \text{DAQ} \rightarrow \text{LabVIEW VI} \rightarrow \text{Plots and Data}$$

Below is a screenshot of the VI. You are encouraged to look up what each block does.



The block diagram shown implements the complete data acquisition and recording pipeline for the vibrating beam experiment. While students are not expected to program LabVIEW in this course, understanding the flow of information through the VI helps clarify how measurements are generated and stored. The VI performs five major operations:

- Initialize acquisition timing
- Acquire sensor data from the DAQ
- Condition and filter signals
- Display and combine signals
- Save measurements to file
- Data flows generally from left to right in the diagram.

A Time Delay block sets a delay before measurement begins. Measurement duration is set using a measurement time input and sampling rate. The system computes how many samples must be collected based on: Sampling rate and Measurement duration. This ensures signals are recorded for the correct time window.

Two DAQ blocks collect data:

- Analog DAQ

- This block acquires voltages from: Strain gauge, Accelerometer, LVDT input, LVDT output. These signals arrive as analog voltages.
- The digital DAQ reads encoder signals. The encoder provides digital pulse data representing displacement.

Signal Routing and Conditioning. After acquisition, signals are separated and routed: Signals are split into individual channels. Signals are sent to indicators and processing blocks. The strain gauge signal passes through a filter block, producing: Raw signal, Filtered signal; Filtering helps remove high-frequency noise. We utilize a 2nd order Butterworth filter to get a steeper roll-off.

Several **signal processing steps** occur: Time-step calculation–The VI computes the sampling interval dt which is needed for later analysis such as differentiation or integration. **Channel alignment and combination**. Signals are grouped so they can be: Displayed together, Saved together, Compared across sensors. **Visualization**. Signals feed into plots, including: time histories, combined signals, Lissajous plots comparing coil motion and displacement. These plots help visually inspect motion and phase relationships.

Metadata and signal formatting. The VI uses blocks labeled Set Dynamic Data Attributes to attach metadata such as: Signal names, Units, Channel organization. This helps LabVIEW interpret signals properly when displayed or saved.

Section 2 Questions

1. From the lab handout, specifically the DAQ channels, are all the inputs single-ended or differential, or a mix?
2. Which channels are analog and which are digital?
3. Why might the encoder be routed to `PFI` lines instead of an analog input?
4. What is the practical difference between single-ended and differential measurement, and why might LVDT be differential?
5. What are the five main operations performed by the LabVIEW VI during measurement?
6. Why must measurement time and sampling rate be specified before acquisition begins?
7. Which sensors provide analog signals and which provide digital signals to the DAQ?
8. Why is filtering applied to some sensor signals before analysis?
9. Why is it useful to display signals live while measurements are being taken?
10. Why are signals grouped and formatted before being saved to file?
11. What is the purpose of saving measurement data instead of relying only on live plots?
12. Explain, in one sentence, the complete path from beam motion to stored measurement data.

3 Dynamics and Modeling

For this lab, the beam is treated as a **cantilever** with a **lumped tip mass model** (beam body mass negligible; head acts as a point mass at the free end). This assumption is stated in the lab for the frequency/flexural rigidity part and should also guide your damping model. Let us start first by deriving the natural frequency formula.

3.0.1 Free Motion of Undamped SDOF (Single Degree-of-freedom) Systems

The motion of a horizontal vibration can be described by Newton's second law , which states that the force acting on an object equals the object mass, m, times its acceleration,

$$\ddot{x} = \frac{d^2x}{dt^2}$$

For the single DOF system this leads to the equation of motion (EoM)

$$m\ddot{x} = -kx \implies m\ddot{x} + kx = 0$$

This is a second-order ordinary differential equation for which a harmonic solution can be assumed:

$$x(t) = A\sin(\omega_n t) + B\cos(\omega_n t),$$

where ω_n is the natural frequency of the system in rad/s. Differentiating twice gives

$$\begin{aligned} \dot{x}(t) &= \omega_n\big(A\cos(\omega_n t) - B\sin(\omega_n t)\big)\,, \\ \ddot{x}(t) &= -\omega_n^2\big(A\sin(\omega_n t) + B\cos(\omega_n t)\big) = -\omega_n^2 x(t). \end{aligned}$$

Thus, displacement $x(t)$, velocity $\dot{x}(t)$, and acceleration $\ddot{x}(t)$ are linked through the natural frequency ω_n. For harmonic motion, the velocity lags displacement by 90°, and acceleration lags displacement by 180°. Substituting $\ddot{x}(t) = -\omega_n^2 x(t)$ into the EoM gives

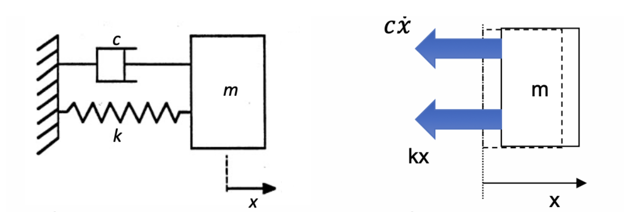
$$-m\omega_n^2 x + kx = 0.$$

For nonzero motion, this yields the natural frequency

$$\omega_n = \sqrt{\frac{k}{m}} \quad \text{[rad/s]} \implies f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{[Hz]}$$

3.0.2 Free Motion of Damped SDOF (Single Degree-of-freedom) Systems

Real vibrating systems often have a source of energy dissipation which can be represented by a massless viscous damper as shown in the figure below. If the mass, m, is moved to one side by x, then the damper produces a drag force, F_d, opposing the motion, which is proportional to the velocity of the mass.



The free body diagram in Figure 2.1b) shows that the damper results in an additional restoring force on the mass of $-c\dot{x}(t)$, where c is a constant, called the damping coefficient or damper rate. Using Newton's second law leads to:

$$m\ddot{x} = -c\dot{x} - kx$$

and rearranging the terms provides the equation of motion for a damped free system:

$$m\ddot{x} + c\dot{x} + kx = 0$$

It can be observed that now the motion of the system no longer depends only on the acceleration, $\ddot{x}$, and displacement, x, but also on the velocity, $\dot{x}$. It is useful to divide the above equation by mass, m, and to rearrange it slightly:

$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = \ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 0$$

Where

$$\omega_n = \sqrt{\frac{k}{m}}$$

is the "undamped natural frequency" for the undamped system as before, and

$$\zeta = \frac{c}{2\sqrt{km}}$$

is called the "viscous damping ratio". Following the approach from the MATH 1B, the solution for the second order ordinary differential equation can be assumed to be of the form:

$$x(t) = Ce^{\mu t}$$

where C is a constant. Differentiating this solution twice leads to:

$$\dot{x}(t) = \mu Ce^{\mu t} = \mu x \implies \ddot{x}(t) = \mu^2 Ce^{\mu t} = \mu^2 x$$

Substituting this into the 2nd order ODE leads to:

$$(\mu^2 + 2\zeta\omega_n\mu + \omega_n^2)x = 0$$

Since $x = 0$ is a trivial solution (no motion), the auxiliary equation becomes:

$$\mu^2 + 2\zeta\omega_n\mu + \omega_n^2 = 0$$

which is a quadratic equation with two roots α and β:

$$\alpha = -\zeta\omega_n + \frac{\sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} = \left(-\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n$$

$$\beta = -\zeta\omega_n - \frac{\sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} = \left(-\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n$$

These two roots can be real or complex, which implies two possible solutions:

$$x_1(t) = Ae^{\alpha t} \quad \text{and} \quad x_2(t) = Be^{\beta t},$$

where A and B are arbitrary constants that depend on the initial conditions. Since x_1 and x_2 are two independent solutions of the equation of motion, the linear combination of them must also be a solution:

$$x(t) = x_1(t) + x_2(t) = Ae^{\alpha t} + Be^{\beta t} \quad \text{for} \quad \alpha \neq \beta$$

There are now three possibilities for the solution of this equation depending on the value of the damping ratio, ζ, leading to an oscillatory motion, non-oscillatory motion, or critically damped motion.

3.0.3 Oscillatory motion

When $\zeta < 1$ the roots of the equation above are complex:

$$\alpha = \left(-\zeta + i\sqrt{1 - \zeta^2}\right)\omega_n$$

$$\beta = \left(-\zeta - i\sqrt{1 - \zeta^2}\right)\omega_n$$

and the general solution becomes:

$$x(t) = Ae^{(-\zeta + i\sqrt{1-\zeta^2})\omega_n t} + Be^{(-\zeta - i\sqrt{1-\zeta^2})\omega_n t}$$

or:

$$x(t) = e^{-\zeta\omega_n t}\left[Ce^{(i\sqrt{1-\zeta^2})\omega_n t} + De^{(-i\sqrt{1-\zeta^2})\omega_n t}\right]$$

Remembering that:

$$e^{(i\sqrt{1-\zeta^2})\omega_n t} = \cos\Big(\sqrt{1-\zeta^2}\,\omega_n t\Big) + i\sin\Big(\sqrt{1-\zeta^2}\,\omega_n t\Big)$$

and

$$e^{-(i\sqrt{1-\zeta^2})\omega_n t} = \cos\Big(\sqrt{1-\zeta^2}\,\omega_n t\Big) - i\sin\Big(\sqrt{1-\zeta^2}\,\omega_n t\Big)$$

The expression above can be re-written in terms of sin and cos:

$$x(t) = e^{-\zeta\omega_n t}\left(A\cos\Big(\sqrt{1-\zeta^2}\,\omega_n t\Big) + B\sin\Big(\sqrt{1-\zeta^2}\,\omega_n t\Big)\right)$$

It can be seen that the exponential term $e^{-\zeta\omega_n t}$ determines the decay rate and the rest of the equation determines the oscillation of vibration. The expression

$$\sqrt{1-\zeta^2}\,\omega_n = \omega_d$$

is the damped natural frequency, ω_d. It should be noted, that for small damping ratio, ζ, the effect of damping on the natural frequency is very small, and can often be neglected. If there is no damping present in the system, $\zeta = 0$, the expression above simplifies to:

$$x(t) = A\cos(\omega_n t) + B\sin(\omega_n t),$$

which was the assumed solution in Chapter 1 for the undamped free vibration of a single DoF system. Using the definition of the damped natural frequency, the expression above can then be written as:

$$x(t) = e^{-\zeta\omega_n t}\left(A\cos(\omega_d t) + B\sin(\omega_d t)\right)$$

which represents the free decaying motion of the damped single DOF system.

3.0.4 Non-oscillatory motion

When the damping ratio $\zeta > 1$ then the roots of the equation above become real and the solution can be written as:

$$x(t) = Ae^{(-\zeta+\sqrt{\zeta^2-1})\omega_n t} + Be^{(-\zeta-\sqrt{\zeta^2-1})\omega_n t}$$

In this case the displacement will not oscillate, but will gradually decay to zero due to the high damping.

3.0.5 Critically damped motion

For the special case when the damping ratio $\zeta = 1$, the damping coefficient will be equal to the critical damping ratio $c = 2\sqrt{km}$, and the roots of the equation above are equal:

$$\alpha = \beta = -\omega_n$$

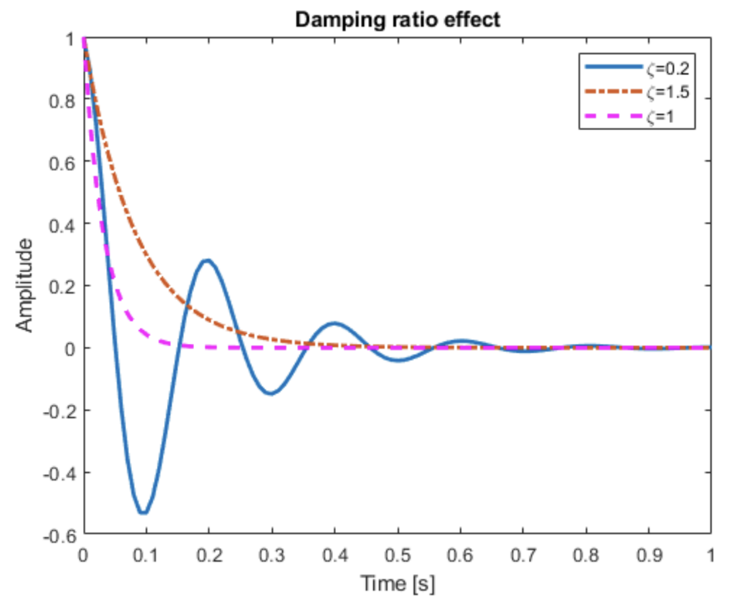
The general solution then becomes:

$$x(t) = (A + Bt)e^{-\omega_n t}$$

As for $\zeta > 1$, the displacement will gradually decay to zero, the rate of decay being a maximum in the case of critical damping.

3.0.6 Comparison of different damping behaviour

Figure below shows the effect of different damping ratios on the vibration response. When $\zeta < 1$ an oscillatory motion is present, where the period of the oscillation corresponds to the damped natural frequency ω_d. Due to the presence of damping, the amplitude gradually decays away, until the system comes to a standstill at its original equilibrium position. If the system is critically damped, $\zeta = 1$, it returns to its equilibrium position very quickly without any overshoot, and when the system is over critically damped $\zeta > 1$ then it will still return to its equilibrium position without any overshoot, but it will take somewhat longer to reach this state.



Damping in a system is considered a very important feature, since it removes kinetic energy from the system over time, and consequently leads to a reduction in vibration amplitude, which in turn can lead to a reduction in noise emitted from a vibrating structure and less stresses induced due to the vibration. Most engineering structures only have a relatively low level of damping ($\zeta < 0.1$) and consequently any vibration will decay away slowly. Damping values close to critical or above are rare in engineering applications, with doors being the main exception. The task of the door damper is to bring the door back to its closed "equilibrium" state as quickly as possible, without causing an overshooting oscillation, which would lead to the door banging into the frame.

Given the large impact damping can have on the response of a structure, determining the type of damping and the amount of damping is of major requirements for an accurate vibration analysis.

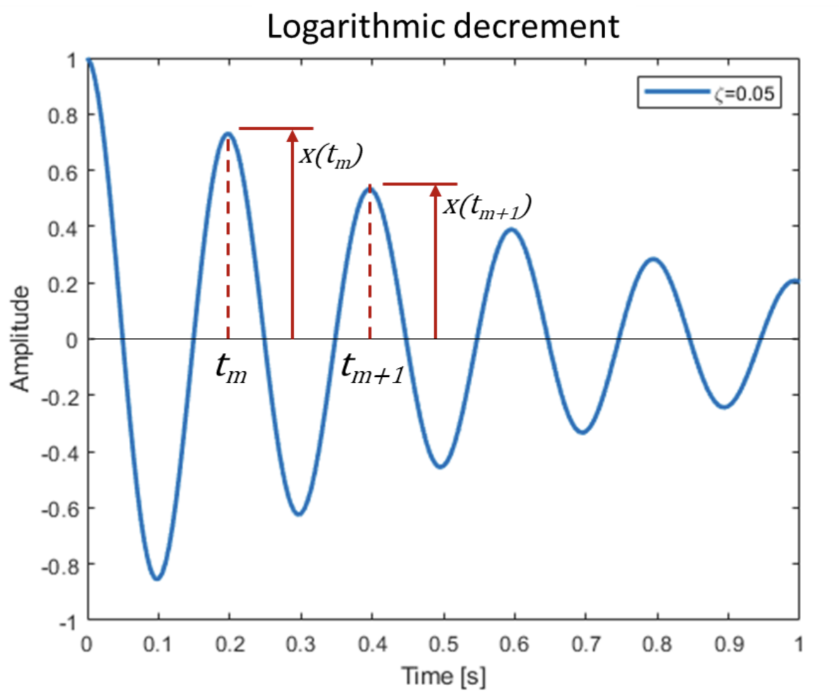
3.0.7 Logarithmic Decrement

The oscillatory solution for a free damped motion is given in the expression above. It can be re-written as:

$$x(t) = Ce^{-\zeta\omega_n t}\sin(\omega_d t + \varphi)$$

where $\tan\varphi = \frac{A}{B}$ represents the phase, φ, between the cosine and sine component of the signal. Using the expression above, the ratio of the amplitudes of successive peaks at time t_m and t_{m+1} in the figure below is given by:

$$\frac{x(t_m)}{x(t_{m+1})} = e^{-\zeta\omega_n(t_m - t_{m+1})}$$



But, $-(t_m - t_{m+1}) = (t_{m+1} - t_m)$, which is the period,

$$t_0 = \frac{2\pi}{\omega_d},$$

of the damped oscillation, so:

$$\frac{x(t_m)}{x(t_{m+1})} = e^{\frac{2\pi\zeta\omega_n}{\omega_d}}$$

With the damped natural frequency,

$$\omega_d = \sqrt{1-\zeta^2}\,\omega_n,$$

the change in vibration amplitude due to damping from one period to the next becomes:

$$\frac{x(t_m)}{x(t_{m+1})} = e^{\left(\frac{2\pi\zeta}{\sqrt{1-\zeta^2}}\right)}.$$

Applying the natural logarithm to the expression above finally gives the logarithmic decrement, δ:

$$\delta = \ln\left(\frac{x(t_m)}{x(t_{m+1})}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}.$$

or for small damping ratios, $\zeta < 0.1$,

$$\delta \approx 2\pi\zeta \qquad \text{or} \qquad \zeta \approx \frac{\delta}{2\pi}$$

The logarithmic decrement, δ, can be directly measured from the vibration signal in the figure above by creating the ratio of two consecutive peak amplitudes,

$$\frac{x(t_m)}{x(t_{m+1})},$$

and taking the natural logarithm of it. For the shown vibration signal this results in:

$$\delta = \ln\left(\frac{0.73}{0.53}\right) = 0.32 = 2\pi\zeta \quad \rightarrow \quad \zeta = \frac{\delta}{2\pi} = 0.05.$$

More generically, if the amplitudes of the m^{th} and $(m+N)^{th}$ peaks are measured, the log of their ratio is:

$$\ln\left(\frac{x(t_m)}{x(t_{m+N})}\right) = \frac{2\pi N\zeta}{\sqrt{1-\zeta^2}} = N\delta$$

or for small damping ratios, $\zeta < 0.1$,

$$\zeta \approx \frac{1}{2\pi N}\ln\left(\frac{x(t_m)}{x(t_{m+N})}\right)$$

This generic form is very useful, since it allows to extract the damping from any free vibration decay of a single DOF system.

3.0.8 Quality factor and damping

Another commonly used measure of damping in dynamic systems is the **quality factor**, Q, which is dimensionless and provides an intuitive measure of how lightly damped a system is.

For a single degree-of-freedom mass–spring–damper system,

$$Q = \frac{\sqrt{mk}}{c} = \frac{1}{2\zeta},$$

so systems with small damping ratio ζ have a large quality factor, meaning oscillations persist for many cycles before dying out.

The quality factor is also related to the sharpness of the resonance peak in the frequency response:

$$Q = \frac{\omega_d}{\Delta\omega},$$

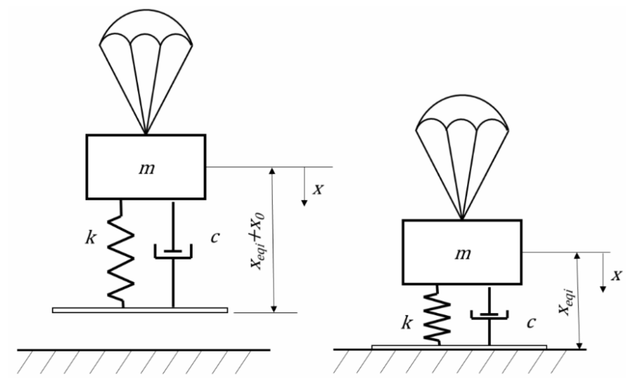
where $\Delta\omega$ is the *full width at half maximum* (FWHM) of the resonance peak. A high-Q system therefore has a narrow resonance peak, while a heavily damped system has a broad peak.

Qualitatively, Q represents the ratio of energy stored in the system to the energy lost per cycle of oscillation. A useful rule of thumb is that Q roughly indicates how many oscillation cycles occur before the motion significantly decays.

In lightly damped mechanical systems, Q is typically large, while systems designed to suppress motion, such as door dampers or vibration isolators, have small Q values corresponding to strong damping.

Section 3 Questions

1. Re-derive the free decaying motion of the damped single DOF system to model the vibrating beam. You should do this succinctly.
2. In an effort to practice doing the math, here is a scenario for you to consider. A radio transmitter and its power supplies are to be dropped by parachute to a polar expedition. To protect the electronics from excessive dynamic loading, the package, which is of mass, $m = 20$ kg, is mounted on a spring-damper system as shown below.



The spring has a stiffness, $k = 10$kN/m, the damper has a damper rate,$c = 540$ Ns/m, and the base plate has negligible mass. If the terminal velocity at touch down of the parachute, $\dot{x} = 8$m/s, what is the maximum distance which the spring is compressed, assuming that the ground on which the package lands is rigid?

4 Experimental Design

One of your tasks in lab will be to implement and execute a plan that extracts ω_n and ζ from free-decay measurements. The lab already tells you what data you will collect, you must now propose the method and justify it.

Section 4 Questions

1. Pick one calibrated signal as your primary $x(t)$-like waveform for damping estimation. Explain your choice in 2-3 sentences.
2. Provide two methods one in the time domain and the other in the frequency domain to estimate the natural frequency ω_n.
3. Provide a method to estimate the damping ratio ζ? Hint: log-decrement.
4. Uncertainty plan, what errors will you have to propagate?
5. In this lab you will collect vibration data from multiple experimental trials. Before any analysis can be performed, the raw data must be loaded, cleaned, and converted into physical units. Write your own MATLAB script that performs the following preprocessing steps:
   (a) Load data from multiple trials stored in separate files.
   (b) Combine all trials into one dataset for analysis.
   (c) Shift the encoder signal so that displacement starts at zero for each trial.
   (d) Convert encoder counts into displacement in meters using the conversion factor provided in the lab manual.
   (e) Save the processed dataset so it can be loaded later without repeating preprocessing.

   To help your script run smoothly during the lab, follow these additional guidelines:

   (a) Rename your data files for the 10 calibration runs using filenames of the form `lab3_calibration[TRIAL_NO].xlsx`.

   For example, the fourth run should be named `lab3_calibration4.xlsx`. This allows your script to automatically load each trial. Alternatively, you may modify your script to match your own filenames.

   (b) Ensure the encoder signal is zeroed for each trial before combining datasets, using the method discussed earlier.
   (c) Convert encoder measurements from counts to meters using the calibration factor provided in the lab instructions.
   (d) After filling in your script, run it to verify that the processed dataset is successfully generated.

   Your script does not need to be long or complex; the goal is simply to ensure you understand how raw data is prepared before analysis.

   The reason we perform this preprocessing is that MATLAB can load saved workspace data much faster than repeatedly reading large matrices from `.xlsx` files. This speed difference becomes significant when working with the hundreds of thousands of data points collected during calibration runs.

   Submit:

   - Your MATLAB preprocessing script.
   - A short explanation (2–3 sentences) describing why zeroing and unit conversion are necessary before analyzing experimental data.

