

HW3

October 2, 2025 at 10:20 am

Reminder: Submit your assignment on Gradescope by the due date. Submissions must be typeset. Each page should include work for only one problem (i.e., make a new page/new pages for each problem). See the course syllabus for the late policy.

While collaboration is encouraged, please remember not to take away notes from any labs or collaboration sessions. Your work should be your own. Use of other third-party resources is strictly forbidden.

Please monitor Ed discussion, as we will post clarifications of questions there.

Problem 1

Give, without proof, a CFG that generates each of the following languages

1. $L_1 = \{w \in \{0,1\}^* \mid w = w^R \text{ and } |w| \text{ is even}\}$ (w^R denotes the reverse of w)
2. $L_2 = \{w \in \{0,1\}^* \mid \text{in every prefix of } w \text{ the number of 0's is no more than the number of 1's}\}$

Problem 2

Design a *deterministic* push-down automaton for the language $L = \{w \in \{a,b\}^* \mid w \text{ starts and ends with the same symbol}\}$ and provide a proof of correctness. (note: the empty string should be accepted)

Problem 3

For an alphabet $\Sigma = \{a,b\}$, show that the language consisting of strings that have an equal number of a's and b's but no substring of the form *abba* is context free. (Hint: you may assume that the language of all strings with no substring *abba* is regular).

Problem 4

For any language A , let $PREFIX(A) = \{v \mid vu \in A \text{ for some non-empty string } u\}$, i.e. the set of proper prefixes of strings in A . Show that the class of context-

free languages is closed under the *PREFIX* operator.